

# Improving the Resolution of the Isotropic Seismic Moment Tensor using Rotational Ground Motions

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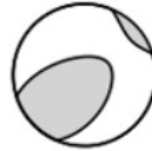
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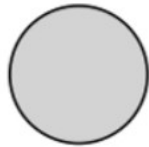
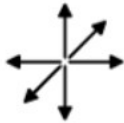


**Seismic moment tensors** are a great tool **to discriminate** whether a seismic source has **explosive** character **or not**.

$$\begin{pmatrix} -0.40 & -0.54 & 0.53 \\ -0.54 & -0.30 & -0.24 \\ 0.53 & -0.24 & 0.70 \end{pmatrix}$$

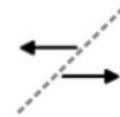


$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

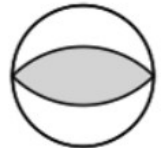


versus

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



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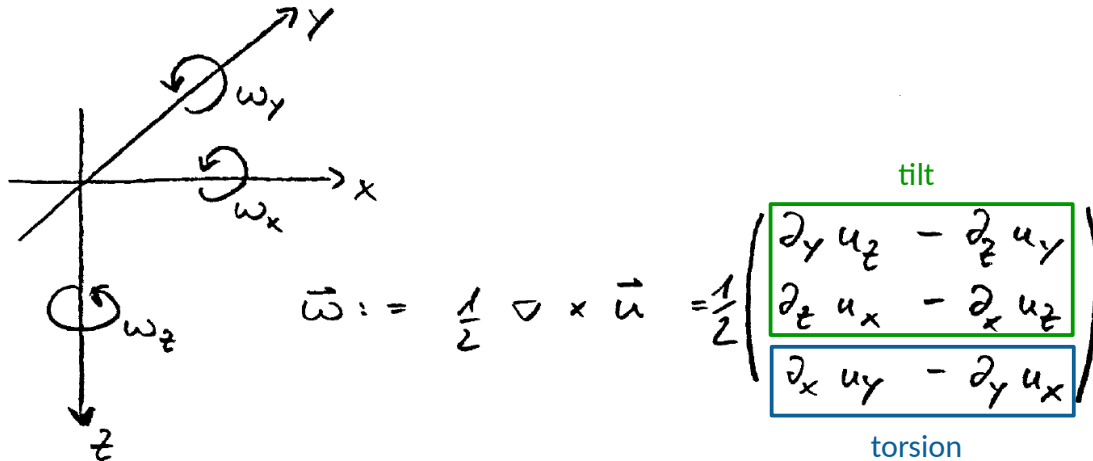
Seismic moment tensors provide information not only about the geometry of a seismic source (tectonic – DC – part) but also with non-tectonic information such as volume changes (isotropic – ISO – part). This feature is crucial to discriminate an explosive source from others, which can hint to a nuclear test. However, that part is often not very well resolved by standard methods. Measuring rotational ground motions might help to obtain more reliable results.

Six components of ground motion are needed to entirely describe the seismic wave-field, three translational and three rotational. Just recently, portable rotation sensors dedicated for seismological applications are available. In previous studies, we show that by inverting both ground motions together, the resolution of the moment tensor can be improved significantly.

In a synthetic set-up for the Korean peninsula we analysed the 2013 Mw5.8 nuclear test of the Democratic People's Republic of Korea. Applying a Bayesian inversion method, we tested three frequency bands. We also tested the inversion with Green's functions based on one- and three-dimensional structural models. The reliability of the source mechanism benefits from both, the three-dimensional structure and rotations, even more in the higher frequency ranges. Thus, also the reliability of the ISO part is increased.



Rotational ground motion  $\vec{\omega}$  is composed by space derivatives of translational ground motion  $\vec{u}$ .



$$\vec{\omega} := \frac{1}{2} \nabla \times \vec{u} = \frac{1}{2} \begin{pmatrix} \partial_y u_z - \partial_z u_y \\ \partial_z u_x - \partial_x u_z \\ \partial_x u_y - \partial_y u_x \end{pmatrix}$$

tilt  
torsion

Rotations provide additional information on the vertical displacement gradient.

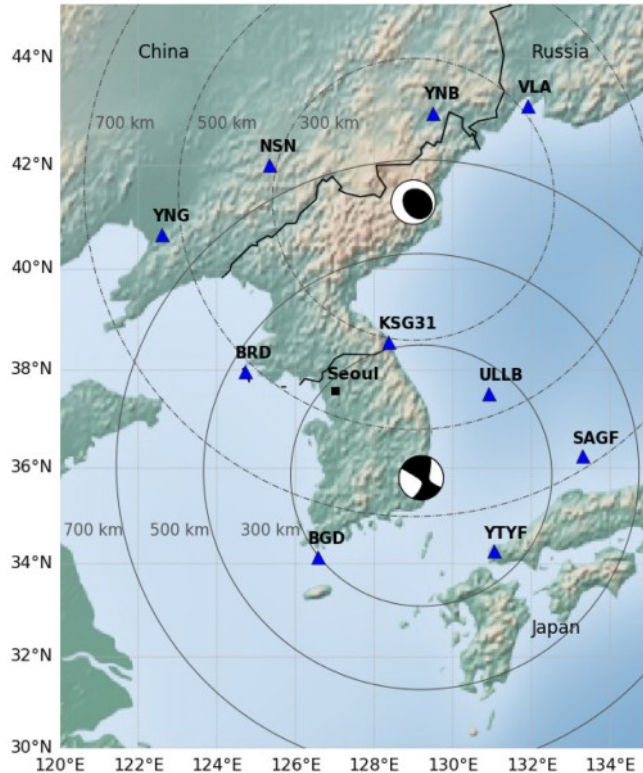
These information are not available from conventional arrays on the Earth's surface.

Since recently portable instruments developed for seismology are available. They measure rotations in a broad frequency range with high sensitivity.

*Schmelzbach et al. 2018*

*Bernauer et al. 2021*

*Izgi et al. 2021*



- ◆ Setting at Korean peninsula:
- ◆ Green's functions calculated with Ses3D ([Fichtner et al. 2009](#))
- ◆ 1- and 3-dimensional velocity models ([Kim et al. 2011, 2016](#))

Mw 5.8 DPRK nuclear test of 2013

DC / ISO / CLVD = 21 / 60 / 19 %

$$\begin{pmatrix} 1.67 & 0.40 & -0.15 \\ & 1.84 & 0.68 \\ & & 3.95 \end{pmatrix}$$

Mw 5.4 Gyeongju, ROK, earthquake of 2016

DC / ISO / CLVD = 88 / 0 / -12 %

$$\begin{pmatrix} 3.58 & 3.21 & 1.67 \\ & -4.11 & 0.60 \\ & & 0.53 \end{pmatrix}$$

*Donner et al. 2020*



$$\begin{array}{c} \text{translation} \\ \vdots \\ \text{rotation} \end{array} \begin{pmatrix} u_1^{rec1} \\ u_2^{rec1} \\ \vdots \\ u_k^{rec1} \\ u_1^{rec2} \\ u_2^{rec2} \\ \vdots \\ u_k^{rec2} \\ \vdots \\ \omega_1^{rec1} \\ \omega_2^{rec1} \\ \vdots \\ \omega_k^{rec1} \\ \omega_1^{recN} \\ \omega_2^{recN} \\ \vdots \\ \omega_k^{recN} \end{pmatrix} = \begin{pmatrix} G_{11}^{u_{rec1}} & G_{12}^{u_{rec1}} & G_{13}^{u_{rec1}} & G_{14}^{u_{rec1}} & G_{15}^{u_{rec1}} & G_{16}^{u_{rec1}} \\ G_{21}^{u_{rec1}} & G_{22}^{u_{rec1}} & G_{23}^{u_{rec1}} & G_{24}^{u_{rec1}} & G_{25}^{u_{rec1}} & G_{26}^{u_{rec1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{k1}^{u_{rec1}} & G_{k2}^{u_{rec1}} & G_{k3}^{u_{rec1}} & G_{k4}^{u_{rec1}} & G_{k5}^{u_{rec1}} & G_{k6}^{u_{rec1}} \\ G_{11}^{u_{rec2}} & G_{12}^{u_{rec2}} & G_{13}^{u_{rec2}} & G_{14}^{u_{rec2}} & G_{15}^{u_{rec2}} & G_{16}^{u_{rec2}} \\ G_{21}^{u_{rec2}} & G_{22}^{u_{rec2}} & G_{23}^{u_{rec2}} & G_{24}^{u_{rec2}} & G_{25}^{u_{rec2}} & G_{26}^{u_{rec2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{k1}^{u_{rec2}} & G_{k2}^{u_{rec2}} & G_{k3}^{u_{rec2}} & G_{k4}^{u_{rec2}} & G_{k5}^{u_{rec2}} & G_{k6}^{u_{rec2}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{11}^{\omega_{rec1}} & G_{12}^{\omega_{rec1}} & G_{13}^{\omega_{rec1}} & G_{14}^{\omega_{rec1}} & G_{15}^{\omega_{rec1}} & G_{16}^{\omega_{rec1}} \\ G_{21}^{\omega_{rec1}} & G_{22}^{\omega_{rec1}} & G_{23}^{\omega_{rec1}} & G_{24}^{\omega_{rec1}} & G_{25}^{\omega_{rec1}} & G_{26}^{\omega_{rec1}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{k1}^{\omega_{rec1}} & G_{k2}^{\omega_{rec1}} & G_{k3}^{\omega_{rec1}} & G_{k4}^{\omega_{rec1}} & G_{k5}^{\omega_{rec1}} & G_{k6}^{\omega_{rec1}} \\ G_{11}^{\omega_{recN}} & G_{12}^{\omega_{recN}} & G_{13}^{\omega_{recN}} & G_{14}^{\omega_{recN}} & G_{15}^{\omega_{recN}} & G_{16}^{\omega_{recN}} \\ G_{21}^{\omega_{recN}} & G_{22}^{\omega_{recN}} & G_{23}^{\omega_{recN}} & G_{24}^{\omega_{recN}} & G_{25}^{\omega_{recN}} & G_{26}^{\omega_{recN}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ G_{k1}^{\omega_{recN}} & G_{k2}^{\omega_{recN}} & G_{k3}^{\omega_{recN}} & G_{k4}^{\omega_{recN}} & G_{k5}^{\omega_{recN}} & G_{k6}^{\omega_{recN}} \end{pmatrix} \begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \\ M_6 \end{pmatrix}$$

# of receivers  $\rightarrow N \cdot k$   $\leftarrow$  # of samples per recording

- time-domain waveform inversion
- synthetic waveform data
- Bayesian approach:

posterior pdf      prior probability density function (pdf)

$$\sigma(\mathbf{m}) = k \rho(\mathbf{m}) L(\mathbf{m})$$

Likelihood function

$$L(\mathbf{m}) = k' \exp \left[ - \sum_l \left( \frac{\chi_l(\mathbf{m})}{s_l} \right) \right]$$

- Shannon's measure of information gain:

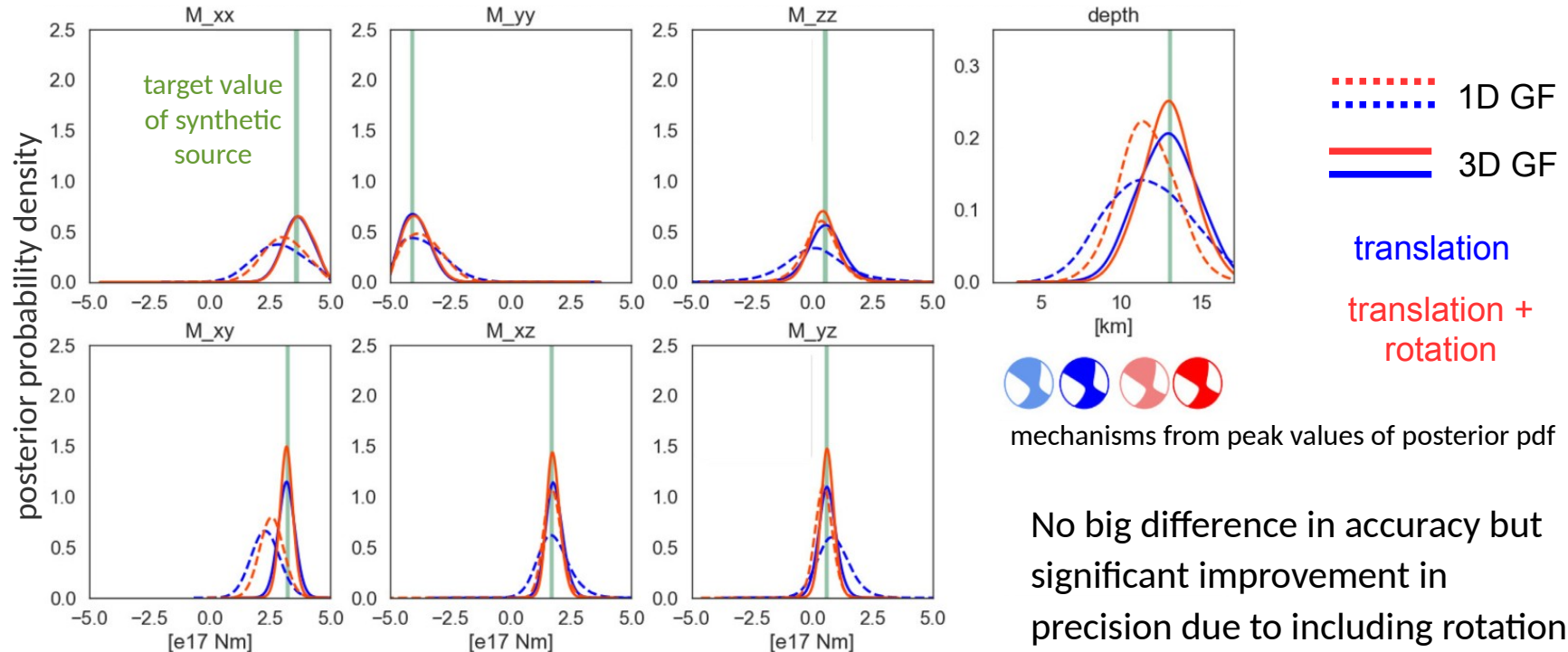
$$I(\rho; \sigma) = \int \rho(x) \log \left[ \frac{\rho(x)}{\sigma(x)} \right] dx \quad (\text{unit: bit})$$

Bernauer et al. 2014

Donner et al. 2016

Donner 2021 (in print)

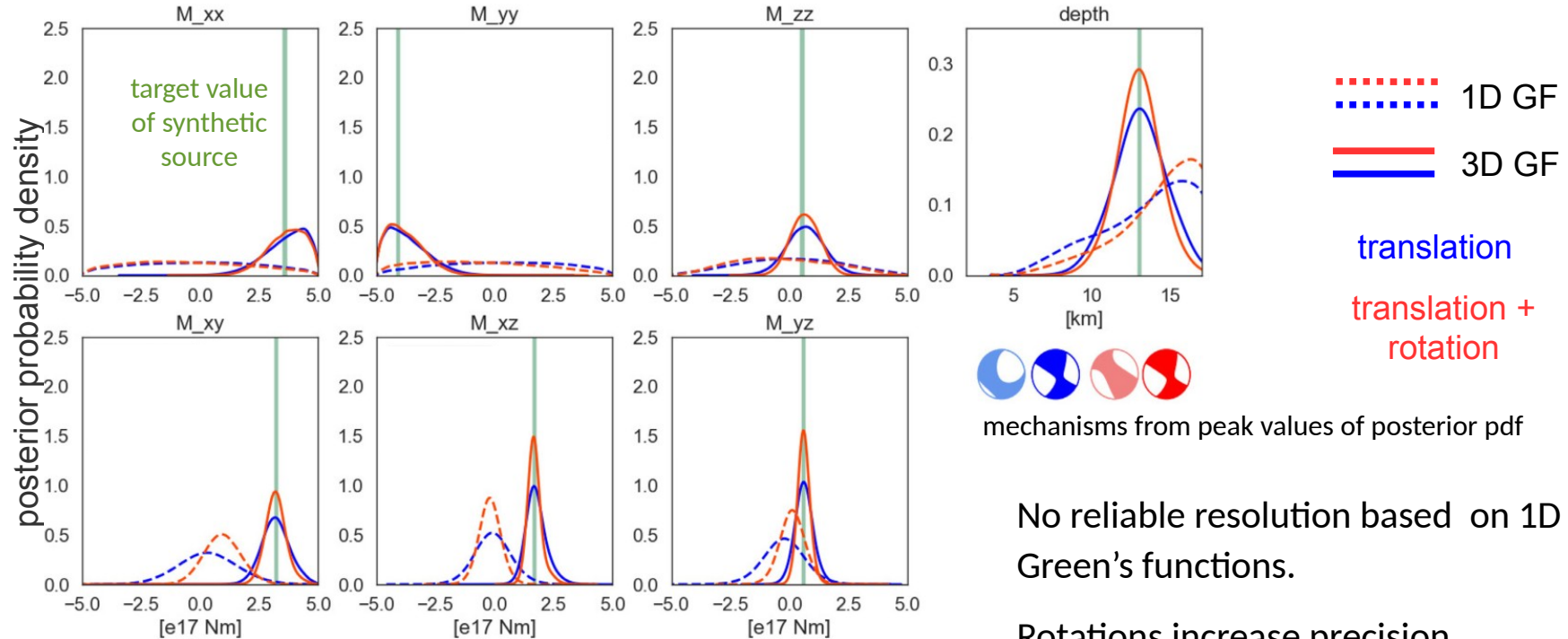
## Synthetic waveform inversion of Gyeongju seismic event; inverted frequencies: 0.02 – 0.05 Hz (20 – 50 sec)



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## Synthetic waveform inversion of Gyeongju seismic event; inverted frequencies: 0.02 – 0.16 Hz (6 – 50 sec)





## Synthetics of Gyeongju seismic event

Rotations alone bring great benefit which can be even increased combined with 3D structure.

0.02 – 0.05 Hz (20 – 50 sec)

Total Information Gain

trans	-----	3C 1D : 9.3 bit	standard
	—————	3C 3D : 14.1 bit	
trans + rot	-----	6C 1D : 12.9 bit	<b>+ 40 %</b>
	—————	6C 3D : 15.8 bit	

0.02 – 0.16 Hz (6 – 50 sec)

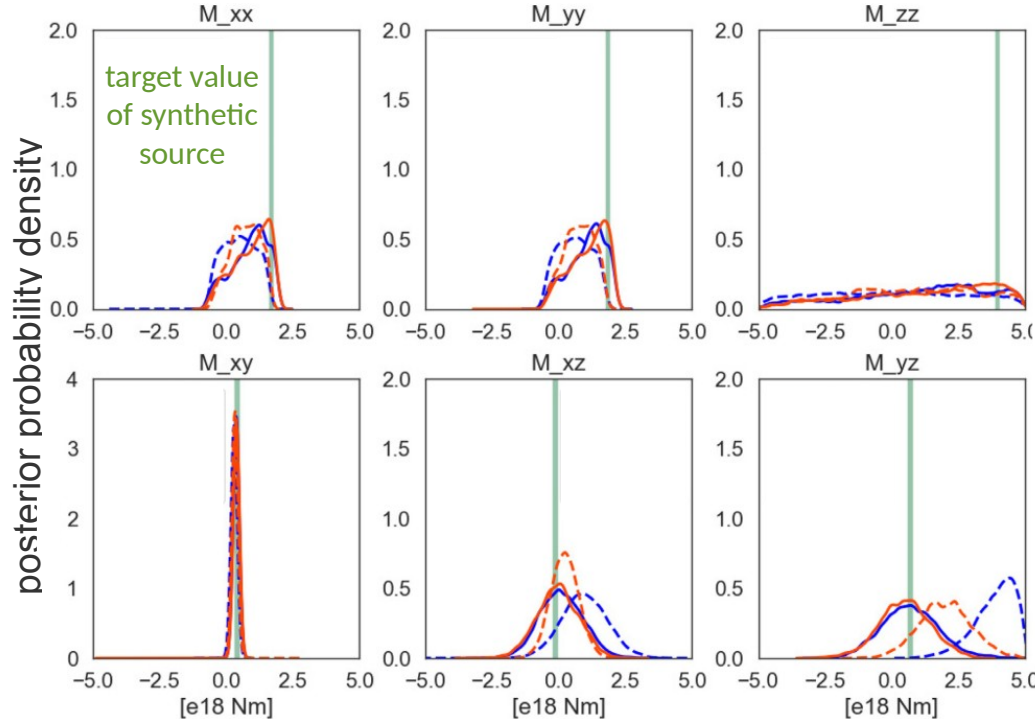
Total Information Gain

trans	-----	3C 1D : 4.6 bit	standard
	—————	3C 3D : 12.1 bit	
trans + rot	-----	6C 1D : 7.1 bit	<b>+ 55 %</b>
	—————	6C 3D : 14.8 bit	

*Donner et al. 2020*

Benefits from 6C even higher for vertically rupturing mechanisms. [Reinwald et al. 2016](#)

## Synthetic waveform inversion of nuclear test; inverted frequencies: 0.02 – 0.05 Hz (20 – 50 sec)



mechanisms from peak values of posterior pdf

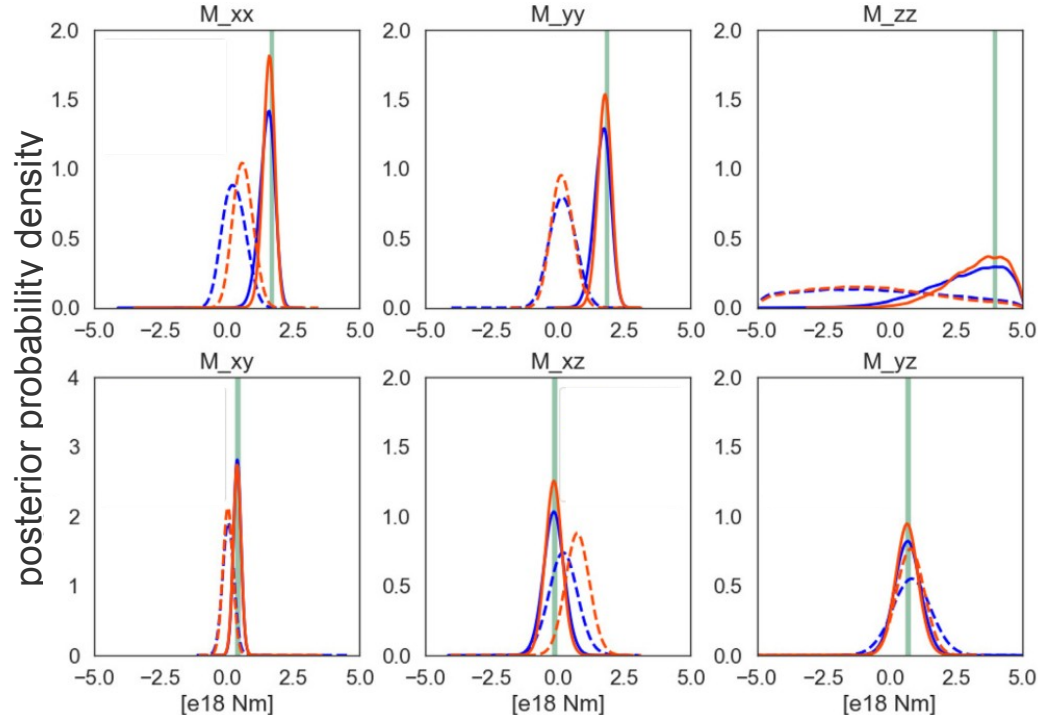
1D GF 3D GF

translation

translation +  
rotation

Especially the ISO part  
(diagonal moment tensor  
elements, first row)  
is problematic, in all cases.

Synthetic waveform inversion of nuclear test; inverted frequencies: 0.02 – 0.16 Hz (6 – 50 sec)



mechanisms from peak values of posterior pdf

1D GF

3D GF

translation

translation +  
rotation

No reliable resolution based on 1D  
Green's functions.

Rotations increase precision.



## Synthetics of nuclear test

Whether 6C or 3D is more supportive depends on the frequency range.

However, both contribute to resolution.

0.02 – 0.05 Hz (20 – 50 sec)

Total Information Gain

trans	-----	3C 1D : 10.7 bit	standard
	———	3C 3D : 10.7 bit	
trans + rot	-----	6C 1D : 11.7 bit	<b>+ 9 %</b>
	———	6C 3D : 11.0 bit	

0.02 – 0.16 Hz (6 – 50 sec)

Total Information Gain

trans	-----	3C 1D : 12.1 bit	standard
	———	3C 3D : 15.4 bit	
trans + rot	-----	6C 1D : 13.5 bit	<b>+ 12 %</b>
	———	6C 3D : 16.7 bit	

*Donner et al. 2020*

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- ◆ Rotational Ground motion are defined as the **curl of the translational wave-field**.
- ◆ They provide additional information on the **vertical displacement gradient**.
- ◆ There is a huge progress in **measurement methods and instrumentation**.
- ◆ The **resolution** of the seismic moment tensor **can significantly be increased**.
- ◆ **Depth-dependent components** have **high potential** to benefit from rotations.
- ◆ Inversion results in same or even better resolution with **less number of receivers**.
- ◆ There is a **high potential to better resolve non-tectonic parts** of the source.
- ◆ **3D structural models** can increase the benefits but have to be taken with care.

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