The Optimised Local Renyi Entropy-Based Shrinkage Algorithm for Sparse TFD Reconstruction

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P3.6-428

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ABSTRACT

• Time-frequency distributions (TFDs) are useful tools for non-stationary signals analysis. However, due to the presence of unwanted cross-terms useful information extraction from TFDs has proven to be a challenging task, especially in the case of noise-corrupted real-life signals. One way to suppress the cross-terms is by employing compressive sensing methods that enforce sparsity in the resulting TFD.

• In this work, we have developed a sparse reconstruction algorithm that reconstructs a TFD from a small sub-set of signal samples in the ambiguity domain. The algorithm utilises the information from both the short-term and the narrow-band Rényi time-frequency entropies, while its parameters are optimised using evolutionary meta-heuristic methods.

• Results are presented for both synthetic and real-life signals in noise, and compared to the state-of-the-art sparse reconstruction algorithms.
Time-Frequency Signal Analysis

- The Wigner-Ville Distribution (WVD) is the most commonly used method for TFD calculation defined as

\[ W_z(t, f) = \int_{-\infty}^{\infty} z(t + \frac{\tau}{2}) z^*(t - \frac{\tau}{2}) e^{-j2\pi ft} d\tau, \]

which introduces wanted components (auto-terms) and highly oscillatory unwanted components (cross-terms).

- The cross-terms can be suppressed in the WVD post-processing by applying a low-pass filter to the ambiguity function (AF):

\[ A_z(\nu, \tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_z(t, f) e^{j2\pi (f\tau - \nu t)} dt df, \]

which leaves the auto-terms positioned at the AF origin and filters out the cross-terms positioned through the rest of the domain:

\[ A_z(\nu, \tau) = g(\nu, \tau) A_z(\nu, \tau), \]

where \( g(\nu, \tau) \) is the AF filter kernel.
The Compressive Sensing (CS) Methods

- The cross-term suppression can be achieved with the sparse reconstruction. Eq. (3) can be rewritten in the matrix form:

\[ \vartheta_z(t, f) = \psi^H \cdot \mathbf{A}_z'(\nu, \tau), \]  

where \( \vartheta_z(t, f) \) is the sparse TFD, or the solution matrix, \( \psi^H \) is the Hermitian transpose of the domain transformation matrix representing the 2D Fourier transform equivalent to (2), and \( \mathbf{A}_z'(\nu, \tau) \) is the CS-AF, or the observation matrix, which is a \( N_{\tau} \times N_{\nu}' \) rectangle containing the AF samples belonging to the auto-terms.

- The rest of the AF is calculated in a way which produces the sparsest TFD. This is an optimization problem with the \( \ell_0 \)-norm-based regularization function:

\[ \vartheta_z^{\ell_0}(t, f) = \arg \min_{\vartheta_z(t, f)} \| \vartheta_z(t, f) \|_0, \]  

subject to: \( \| \vartheta_z(t, f) - \psi^H \mathbf{A}_z'(\nu, \tau) \|_2^2 \leq \epsilon, \)  

where \( \epsilon \) is a user-defined solution tolerance.
METHODS

The Local Rényi Entropies

Figure 1: (a) Short-term Rényi entropy; (b) narrow-band Rényi entropy; (c) both local entropies on one-component signal.

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The Local Rényi Entropy Based Shrinkage Algorithm for Sparse TFD Reconstruction

• The proposed shrinkage algorithm is based on the Two-step iterative shrinkage/thresholding (TwIST) algorithm:

\[
\begin{align*}
\varphi_{z}^{0}(t, f)^{[n+1]} &= (1 - \alpha)\varphi_{z}^{0}(t, f)^{[n-1]} + (\alpha - \beta)\varphi_{z}^{0}(t, f)^{[n]} + \beta \cdot \text{shrink}\left\{ \varphi_{z}^{0}(t, f)^{[n]} + \psi^{H}\left( A^{\tau}(\nu, \tau) - \psi\varphi_{z}^{0}(t, f)^{[n]} \right) \right\},
\end{align*}
\]

where \( \alpha \) and \( \beta \) are user-defined TwIST relaxation parameters.

• \textit{shrink}\{\cdot\} operator is based on the \textbf{short-term} and the \textbf{narrow-band Rényi entropies} which give information on the number of signal components in each time- or frequency-slice, \( N_{c}(t) \) or \( N_{c}(f) \), respectively.
• CS-AF filtering concentrates mainly on the auto-terms; hence, the obtained sparse TFD has auto-terms with larger non-negative energy surface than cross-terms.
• The shrinkage algorithm leaves samples belonging to $N_c(t)$ or $N_c(f)$ largest surfaces in time- or frequency-slice.
• Parameters $\delta_t / \delta_f$ control the number of samples left in the final time-/frequency-slice.
• The algorithm performance is controlled by the percentage of utilization of each Rényi entropy information, controlled by the parameter $p$:

$$\zeta_z(t, f) = p \cdot \zeta^t_z(t, f) + (1 - p) \cdot \zeta^f_z(t, f), \quad (7)$$

where $\zeta^t_z(t, f)$ and $\zeta^f_z(t, f)$ are TFDs obtained by the proposed shrinkage performed over time- or frequency-slices, respectively.
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**RESULTS**

Considered Test Signals

\[ z_s - \text{synthetic signal composed of linear and sinusoidal FM components embedded in additive white Gaussian noise with signal-to-noise ratio = 3 dB.} \]

\[ z_r - \text{real-life gravitational signal (https://losc.ligo.org)} \]

\[ \text{The reconstruction performance has been compared to the following state-of-the-art reconstruction algorithms: TwiST, Sparse reconstruction by separable approximation (SpaRSA) and Split augmented Lagrangian shrinkage algorithm (SALSA).} \]

Figure 2: WVD and its respective AF of: (a),(b) \( z_s \); (c),(d) \( z_r \). The automatically selected CS-AF area has been marked by a rectangle.
Parameters Optimization

- We have used the multi-objective optimization method based on the Particle swarm optimization (MOPSO) method; a stochastic optimization algorithm inspired by nature and social behaviour between birds in swarms.

Objectives which need to be minimized:

- mean squared errors between the local number of components (obtained by the short-term and the narrow-band Rényi entropy) in the starting and reconstructed TFDs, $\text{MSE}_{t,f}$ - preserve components resolution and consistency
- the number of regions with continuously-connected AF samples, $N_r$ - preserves components connectivity

For the proposed algorithm, a multi-objective problem is formalized as:

$$\min \{ \text{MSE}_t, \text{MSE}_f, N_r(\alpha, \beta, p, \delta_t, \delta_f) \},$$

subject to $\alpha, p, \delta_t, \delta_f \in [0,1], \beta \in [0, 2\alpha]$.  

(8)
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RESULTS

Figure 3: Reconstructed sparse TFDs of: (a) $z_s$ with the proposed algorithm; (b) $z_s$ with the TwIST algorithm; (c) $z_s$ with the SpaRSA algorithm; (d) $z_s$ with the SALSA algorithm; (e) $z_r$ with the proposed algorithm; (f) $z_r$ with the TwIST algorithm; (g) $z_r$ with the SpaRSA algorithm; (h) $z_r$ with the SALSA algorithm.

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Table 1: Comparison with the state-of-the-art algorithms. The bold values indicate the best performing/fastest reconstruction algorithm.

<table>
<thead>
<tr>
<th>Rényi</th>
<th>TwIST</th>
<th>SpaRSA</th>
<th>SALSA</th>
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<tbody>
<tr>
<td>$p = 0.816$</td>
<td>$p = 1$</td>
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<td></td>
</tr>
<tr>
<td>$\delta_t = 0.010$</td>
<td>$\delta_t = 0.913$</td>
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<tr>
<td>$\delta_t = 0.948$</td>
<td>$\delta_t = 0.823$</td>
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<tr>
<td>$\bar{z}_S$</td>
<td>$\bar{z}_T$</td>
<td>$\bar{z}_S$</td>
<td>$\bar{z}_T$</td>
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<tr>
<td>MSE$_t$</td>
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<td><strong>0.0052</strong></td>
<td><strong>0.0132</strong></td>
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<tr>
<td>MSE$_f$</td>
<td><strong>0.0110</strong></td>
<td><strong>0.0048</strong></td>
<td>0.0423</td>
</tr>
<tr>
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<td>11</td>
<td>18</td>
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<tr>
<td>$t[s]$</td>
<td>0.165</td>
<td>0.381</td>
<td><strong>0.191</strong></td>
</tr>
</tbody>
</table>
CONCLUSIONS

- By utilizing both local Rényi entropies simultaneously, the proposed algorithm reduces inaccuracies of each entropy when analysing signals with components having different FM modulations.

- The proposed algorithm achieves competitive results when compared to the state-of-the-art sparse reconstruction algorithms, providing the best compromise between the objective functions and the algorithm execution time.