An Alternative Proposal for Estimation of Body Wave Magnitude Taking Account of Noise Magnitudes

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Abstract

- We build on Radzyner et al. (2017).
- As there, we consider a modified relationship of station magnitude to log(A/T).
- Our extension is to also include noise magnitudes with maximum likelihood estimation.
- We show further support for the modified relationship.
- We find that there is also need to modify the standard deviation for each station reading.

Introduction

- Body wave magnitude (*mb*) is important for CTBT verification, both to discriminate between earthquakes and explosions, and for yield estimation of a presumed explosion.
- The standard estimate is to average station magnitudes, computed as $m_{ij} = \log(A_{ij} / T_{ij}) + VC_{ij}$.
- The estimate can also include noise magnitudes, computed from the same formula, but with the event signal buried in background noise.
- Radzyner et al. proposed modifying the station magnitudes to $m_{ij} = (1 + a_i)\log(A_{ij} / T_{ij}) + VC_{ij} + b_i$.
- We combine that proposal with estimation that also includes noise magnitudes.

Methods

- Develop appropriate routines for estimating calibration parameters and magnitudes.
- Apply them to REB data.
- Test via simulations.

Notation and Assumptions

- Denote by A_{ij}/T_{ij} the amplitude/period ratio at station i for event *j*. If the event signal is lost in noise, then $A_{ij}/T_{ij} \le A_{ij}^N/T_{ij}^N$ where A_{ij}^N/T_{ij}^N is the matching ratio for the background noise.
- The above ratios translate, via the assumed equations into an observed magnitude m_{ij} or a noise magnitude m_{ij}^N.
- We assume that $m_{ij} \sim N(m_j, \sigma_i^2)$ where m_j is the event magnitude, and $\sigma_i^2 = (1+a_i)^2\sigma^2$.

Methods

- Estimation is by maximum likelihood, summing over all results.
- Log likelihood contribution of an observed magnitude is

$$-\log \sigma_i - 0.5(m_{ij} - m_j)^2 / \sigma_i^2$$

- Log likelihood contribution of a noise magnitude is

$$\Phi\left((m_{ij} - m_j)/\sigma_i\right) \; .$$

- The procedure estimates the magnitudes m_j, the station-specific calibration parameters a_i and b_i and the variance σ^2.

Results: Analysis of REB Data

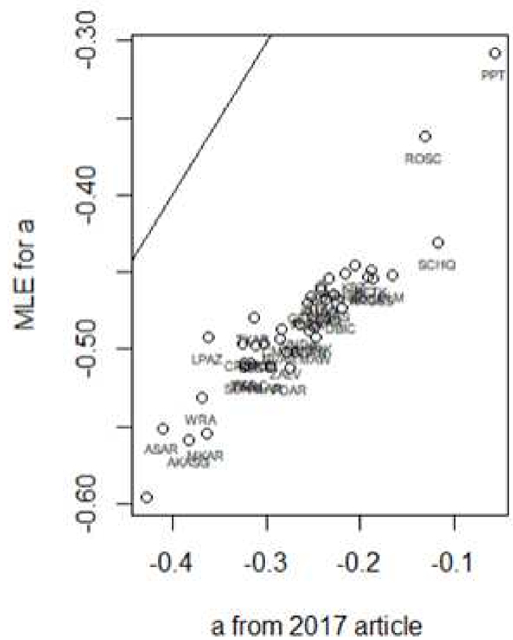
- Events from the period 1.1.2019 to 31.12.2019.
- Includes 39 primary stations.
- There are ~43,000 phase arrivals, of which 35% are observed magnitudes and 65% are noise magnitudes.
- The percent of noise magnitudes varies from 10.5% at station ASAR to 98.5% at station PPT.

Results: Analysis of REB Data

- Estimates typically have a_i=0.4, b_i=-0.1, similar to findings in Radzyner et al. (2017).
- The resulting estimated magnitudes have less spread than the REB magnitudes (computed with MLE); in particular, high magnitudes are reduced.
- However, they are also “more consistent” internally, with less intra-event spread.

Results: Simulation Assessment

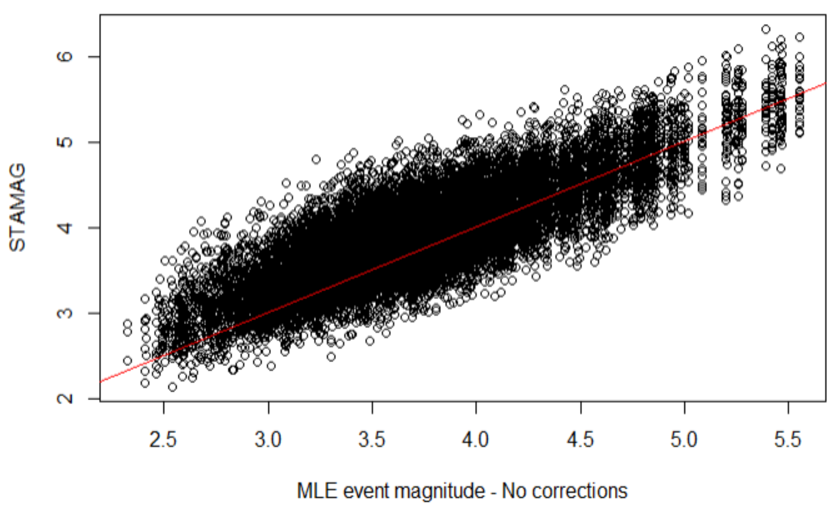
- Data were simulated from the model using the values of a_i from Radzyner et al. (2017).
- Plot of estimated a_i vs generating value.



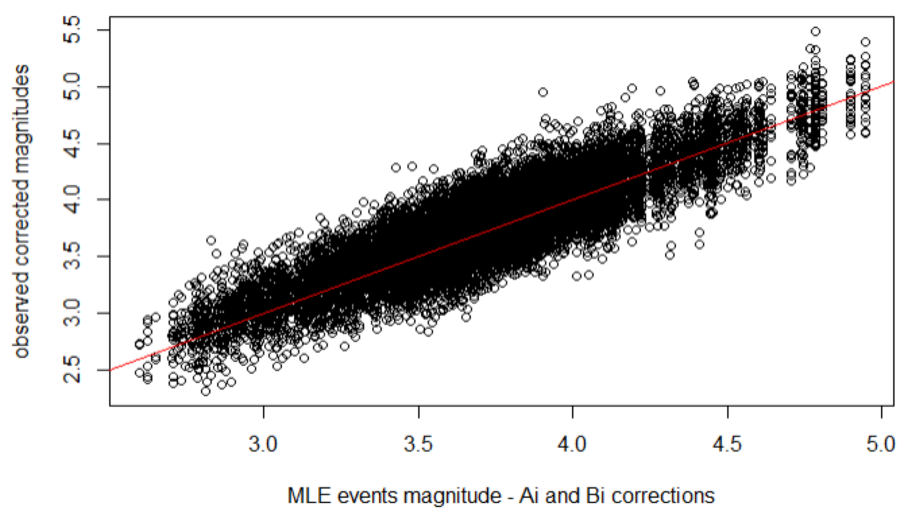
Results: Analysis of REB Data

Plot of event magnitude vs station magnitude for observed magnitudes only; standard formula compared to our estimated model.

Standard formula



Our estimated model



Conclusions

- We extend the approach in Radzyner et al. (2017) to include noise magnitudes.
- This requires appropriate modification of error variances.
- Results are similar to those found earlier.
- High magnitudes are reduced; can be easily corrected by calibration.
- There is greater consistency among reporting stations about event magnitudes.