

Using Nonlinear Thresholding of Stockwell Transforms to Denoise Seismic Waveforms

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INTRODUCTION AND MAIN RESULTS

Stockwell transform (ST) is an extension of the continuous wavelet transform (CWT). In contrast to CWT and short-time Fourier transform, however, ST provides a better time and frequency resolution. We leveraged that advantage by implementing signal enhancement approaches in the ST domain. Thresholding in ST domain appears to be an efficient method for noise suppression and can be used to remove noise that persists after polarization filtering.

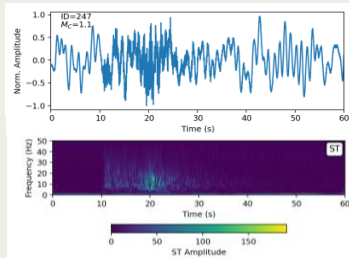
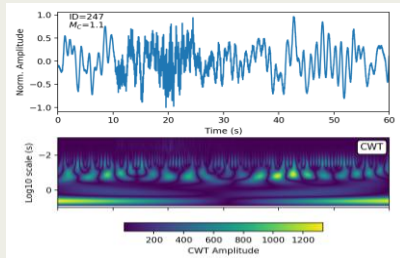


Introduction

- Despite significant progress in seismic data analysis, the separation of signal and noise remains a fundamental problem.
- In the seismology community, frequency filtering (FF) is the most commonly used method for noise suppression.
- When the signal of interest and noise share the same frequency band, noise cannot be filtered out with FF without infringing on the signal of interest.
- Therefore, there is a need for alternative methods for noise suppression.

Objectives

- Stockwell transform (ST) is an extension of the continuous wavelet transform (CWT). In contrast to CWT and short-time Fourier transform, however, ST provides a better time and frequency resolution.



- The objectives of this work is to leverage the advantage in terms of resolution provided by ST by exploring signal enhancement approaches in that domain.

Methods and Data

S Transform

$$S(\tau, f) = \int_{-\infty}^{+\infty} x(t) \frac{|f|}{\sqrt{2\pi}} e^{-\frac{(\tau-t)^2 f^2}{2}} e^{-i2\pi f t} dt$$

f - Frequency
 τ - Time lag or location

ST Thresholding

- Noise is assumed to be stationary throughout the waveform to be processed.
- Pre-event window is used to estimate the frequency dependent threshold
- The thresholded ST coefficient are defined as below (soft thresholding)

$$\tilde{S}(\tau, f) = \begin{cases} \text{sign}[S(\tau, f)] (|S(\tau, f)| - \beta(f)) & \text{if } |S(\tau, f)| \geq \beta(f), \\ 0 & \text{otherwise} \end{cases}$$

in which $\text{sign}[S(\tau, f)] = \frac{S(\tau, f)}{|S(\tau, f)|}$

Polarization Filtering

- Noise is expected to be less polarized than signal.
- Polarization filtering is generally used to separate signal from noise and to improve phase identification.
- Rectilinearity:**

$$RL(t, f) = 1 - \frac{(\lambda_2(t, f) + \lambda_3(t, f))}{2\lambda_1(t, f)},$$

where $\lambda_i(t, f)$ ($i = 1, 2, 3$) are the eigenvalues of the covariance matrix obtained from the 3 orthogonal components of ground motion, from which $\lambda_1(t, f)$ is the largest eigenvalue.

Methods and Data (continued)

- Directionality:** where α , θ , and δ are the angles between the eigenvector associated with $\lambda_1(t, f)$ and the Z, R, and T components, respectively.

$$\begin{cases} D_Z(t, f) = \cos^2(\alpha) \\ D_R(t, f) = \cos^2(\theta), \\ D_T(t, f) = \cos^2(\delta) \end{cases}$$

Polarization Operator

- The operators of the polarization filter are defined as:

$$\begin{cases} P_Z(t, f) = RL(t, f) \cdot D_Z(t, f) \\ P_R(t, f) = RL(t, f) \cdot D_R(t, f), \\ P_T(t, f) = RL(t, f) \cdot D_T(t, f) \end{cases}$$

- Filtering is accomplished as expressed below.

$$\begin{cases} \hat{S}_Z(\tau, f) = S_Z(\tau, f) \odot P_Z(\tau, f) \\ \hat{S}_R(\tau, f) = S_R(\tau, f) \odot P_R(\tau, f) \\ \hat{S}_T(\tau, f) = S_T(\tau, f) \odot P_T(\tau, f) \end{cases}$$

where $S_Z(\tau, f)$, $S_R(\tau, f)$, and $S_T(\tau, f)$ are the STs of Z, R, and T component seismograms, respectively; and $\odot$ denotes the Hadamard product.

Data

The approaches are tested using regional data from the declared 2006 North Korea nuclear explosion recorded in South Korea and local data from the University of Utah Seismograph Stations.

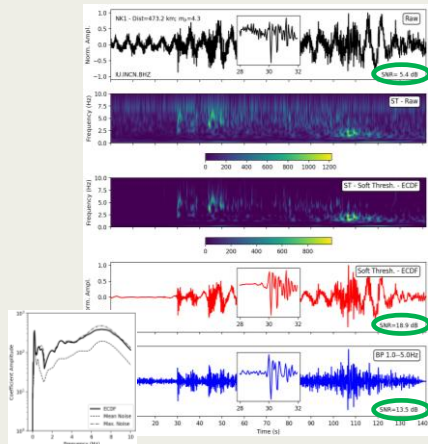


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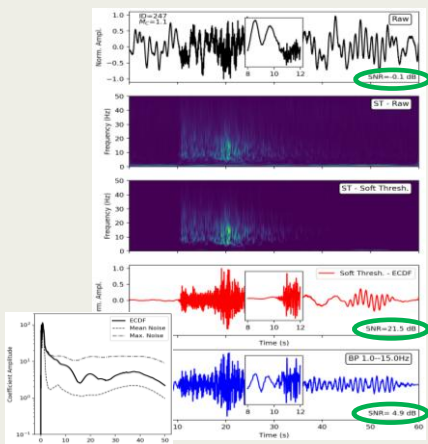
Results

Examples ST Thresholding Regional Data



Data from the 2006 declared North Korea nuclear test recorded at station INCN located in South Korea.

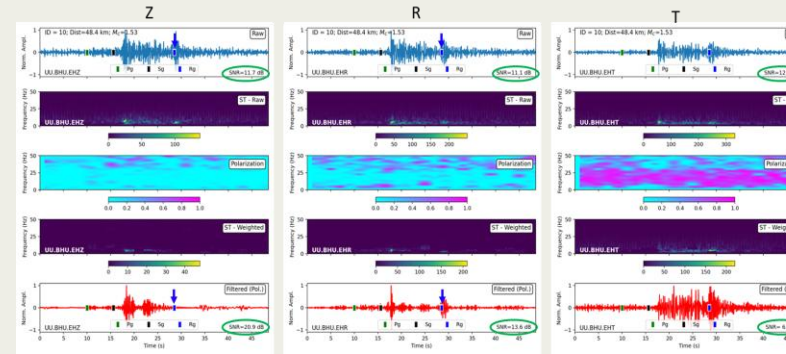
Local Data



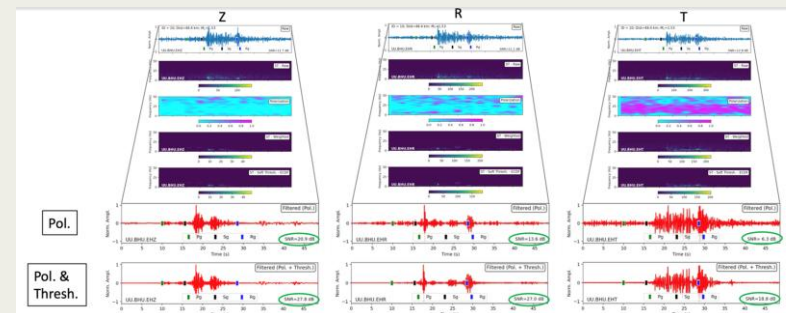
Data from a local event in Utah.

Results (continued)

Example Polarization Filtering

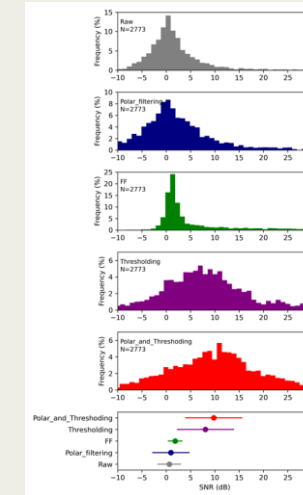


The polarization filter in this case is tailored to improve signals that are linearly polarized. For that reason, Rg signals, which are elliptically polarized, are significantly reduced or fully suppressed. After polarization filtering, some noise still persists.



Noise that persists after polarization filter is removed using ST-thresholding as shown in this figure, further improving the SNR.

Results (continued)



Distribution of SNR for the different approaches for a dataset of 2773 waveforms. The bottom plot shows for each method the median value of SNR and the associated median absolute deviation.

Conclusions

- This work is at its early stage; but based on handful of examples, the approaches are promising.
- Thresholding in ST domain appears to be an efficient method for noise suppression and can be used to remove noise that persists after polarization filtering, improving the SNR further and allowing an effective exploitation of the different phases.
- Future work includes evaluating the implement polarization filtering using synthetic data containing signals with known polarization attributes.