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# Mathematical Morphological Filtering with a Self-adaptive Reconstruction Technique and Application to Local Seismic Data

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## INTRODUCTION AND MAIN RESULTS

Mathematical morphological filtering (MMF) stems from the branch of mathematics referred to as mathematical morphology, which is based on set-theoretic concepts. MMF is efficient for low SNR data, and significantly outperforms frequency filtering (FF) in that SNR range. The results suggest that in an operational setting, MMF cannot replace FF; however, detection can be improved if MMF is used to supplement FF in some scenarios.

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SnT 2025
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## Introduction

- Mathematical morphological filtering (MMF) is a less known method of noise suppression.
- It stems from the branch of mathematics referred to as mathematical morphology, which is based on set-theoretic concepts.
- In contrast to frequency filtering (FF), which operates on frequency information and hence requires Fourier transforms, MMF leverages waveform shape and works entirely in the time domain.

## Objectives

| Noise Suppression Method | Drawbacks |
|---|---|
| Frequency filtering | Inadequate when signal and noise share the same frequency band; causes significant distortion in amplitudes |
| Machine learning denoising | Requires pre-trained model for the region of interest, which necessitates a lot of resources |
| Wavelet-based thresholding | Causes significant distortion in amplitudes; can introduce artefacts in the waveforms |

- The purpose of this work is to improve signals from local-distance events (both earthquakes and explosions) and avoid issues associated with other noise suppression methods.

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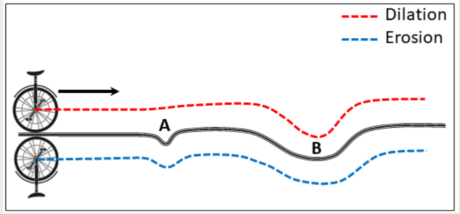
## Methods

**Morphological Operations**

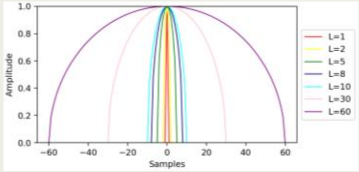
- The key element of morphological operations is the structuring element (SE).
- The shape and size of SE determine the outcome of the morphological operation.
- MMF relies on 2 basic morphological operations (dilation & erosion) and compound operations involving the two.

**Dilation:** $(F \oplus S)(n) = \max\,[F(n-m) + S(m) \mid n \in P, (n-m) \in Q]$

**Erosion:** $(F \ominus S)(n) = \min\,[F(n+m) - S(m) \mid n \in P, (n+m) \in Q]$




The selected SEs are symmetric elliptical, with the general form: $S_{(A,L)}(m) = A\sqrt{1-(m/L)^2}$, where $m \in [-L, L]$, $L > 0$ is the length (in samples), and $A$ the amplitude, with $A = 0.2(\max(|F(n)|))$



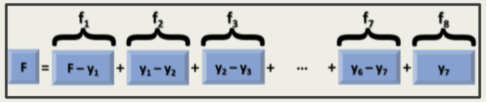

## Methods (continued)

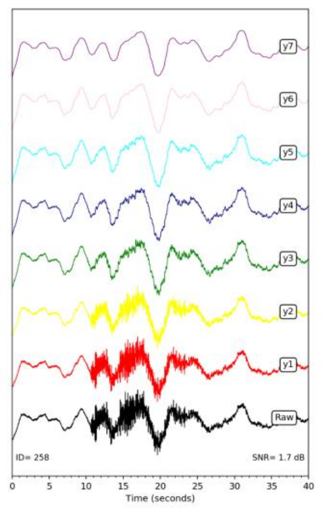
**Multi-scale Morphological Decomposition**

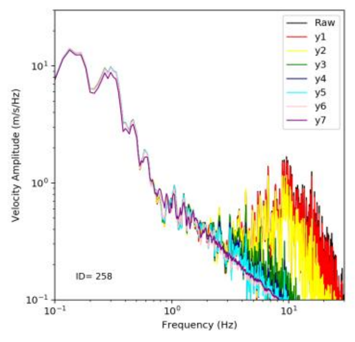
- **Opening:** $(F \circ S)(n) = \big((F \ominus S) \oplus S\big)(n)$
- **Closing:** $(F \bullet S)(n) = \big((F \oplus S) \ominus S\big)(n)$
- **Open-closing:** $(F \sqcup S)(n) = \big((F \circ S) \bullet S\big)(n)$
- **Close-Opening:** $(F \sqcap S)(n) = \big((F \bullet S) \circ S\big)(n)$

$$y_i(n) = \frac{(F \sqcup S_i)(n) + (F \sqcap S_i)(n)}{2}, \; i = 1, \dots, 7$$

$$\begin{cases} f_1(n) = F(n) - y_1(n) \\ f_i(n) = y_{i-1}(n) - y_i(n), \; i = 2, \dots, 7 \\ f_8(n) = y_7(n) \end{cases}$$



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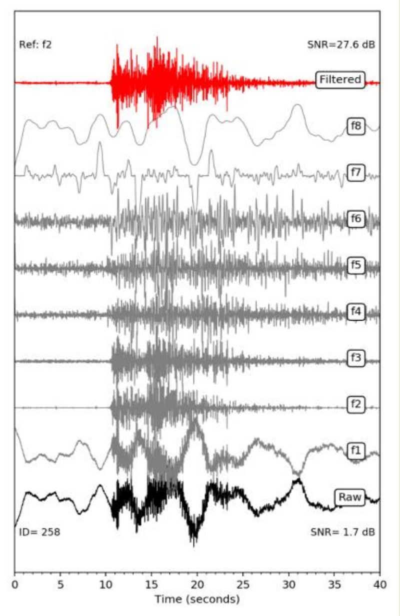
## Methods (continued)

### Morphological Reconstruction & MMF

$$F_{fil}(n) = \sum_{i=1}^{8} \lambda_i f_i\,(n)$$

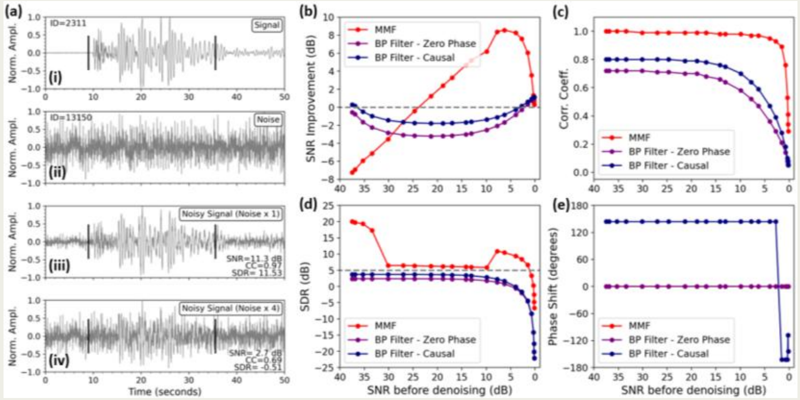
$$\begin{cases} \lambda_{i,ref} = 1, \text{if } CC_{i,ref} \geq 0.2 \\ \lambda_{i,ref} = (SNR_i/SNR_{ref})\left|CC_{i,ref}\right|, \text{if } CC_{i,ref} < 0.2 \end{cases}$$

$$\boxed{F_{fil} = \lambda_1 f_1 + \lambda_2 f_2 + \lambda_3 f_3 + \cdots + \lambda_7 f_7 + \lambda_8 f_8}$$



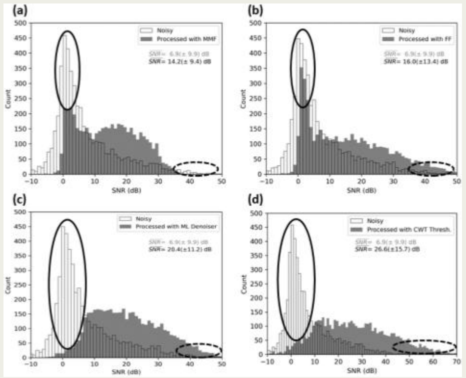

## Results

### Effect of Input Seismogram Quality




MMF is efficient for low input SNR, and significantly outperforms FF in that SNR range.

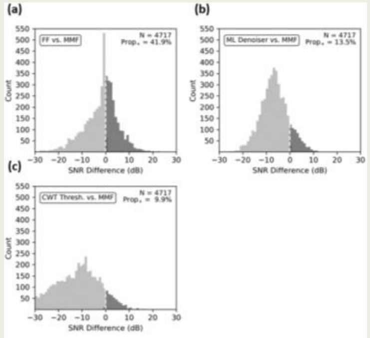
### Comparison with some Established Noise Suppression Methods




Comparison based on a dataset of >4700 constructed noisy waveforms.

- Average improvements in SNR are about 7, 9, 13, and 20 dB for MMF, FF, ML denoising, and CWT thresholding, respectively.
- Compared with FF, however, the other approaches, including MMF improve the SNR for low-quality data, reducing the number of low-SNR waveforms

## Results (continued)




- For ~42% of the dataset, MMF outperforms FF; and the SNR gain achieved by MMF is as high as ~23 dB.
- This proportion is only ~ 10%–14% for ML denoising and CWT thresholding.

## Conclusions

- As noise suppression method, MMF is efficient for low-SNR data and significantly outperforms FF in that SNR range.
- For most of waveforms, FF, ML denoising, and CWT thresholding result in higher SNRs compared with MMF; however, for ~42% of the waveforms, MMF outperforms FF.
- Compared to ML denoising and CWT thresholding, this proportion drops to ~10%–14%.
- The results suggest that in an operational setting, MMF cannot replace the other noise suppression methods; however, detection can be improved if MMF is used to supplement them in some scenarios.
- MMF could help detect signals in problematic low-SNR data, which are currently being missed particularly when using FF alone.

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