



A looped inference approach for more efficient gravitational field mapping during OSI

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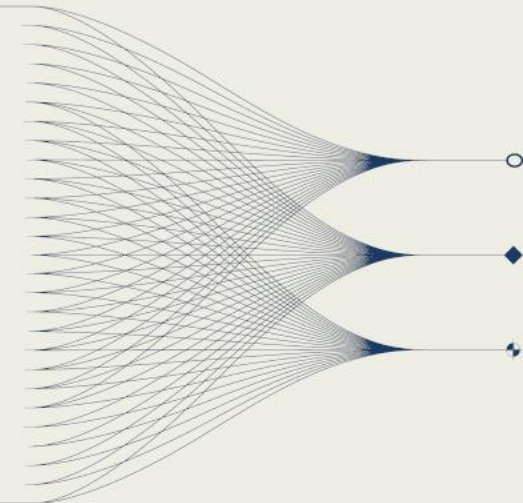
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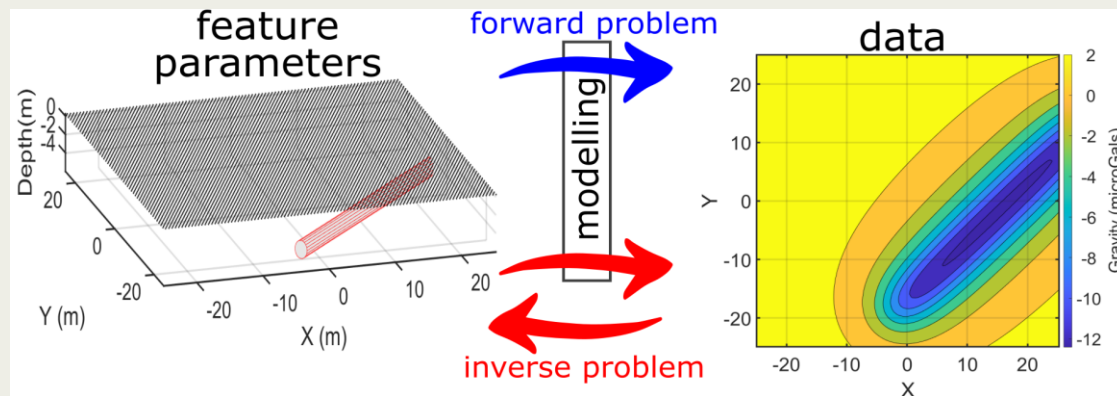
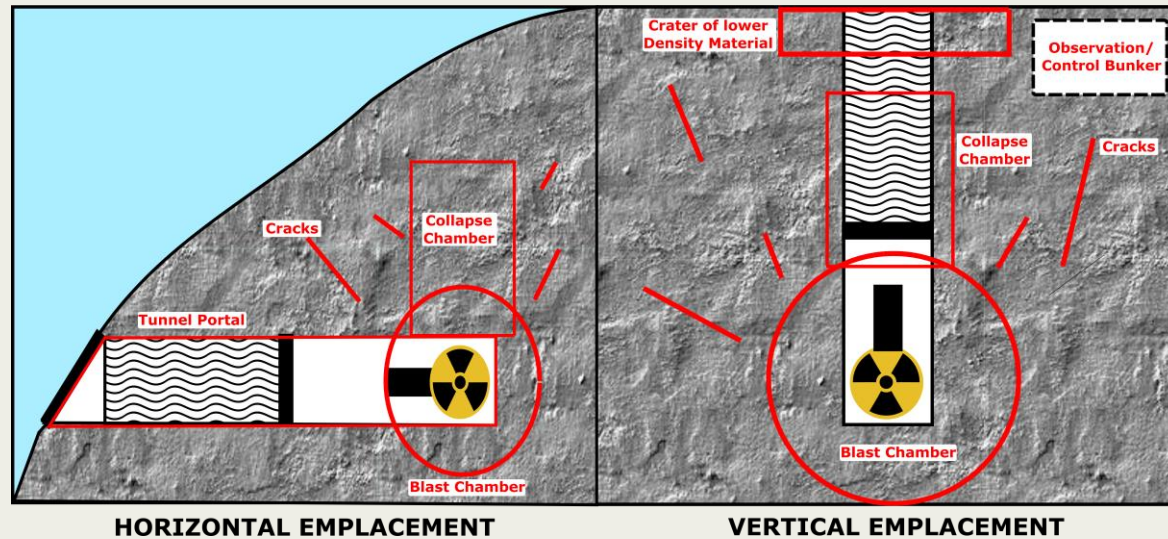
INTRODUCTION AND MAIN RESULTS

Gravitational field mapping is an OSI technique to find tunnel, voids and collapse features, but is slow and spatial resolution sparse leading to high ITF risk. Using an in-field looped Bayesian algorithm selects measurements where information on model parameter variation is maximised, reducing the number of points for a successful outcome. A synthetic example for a tunnel shows improvements in speed and accuracy compared to a standard grid approach.

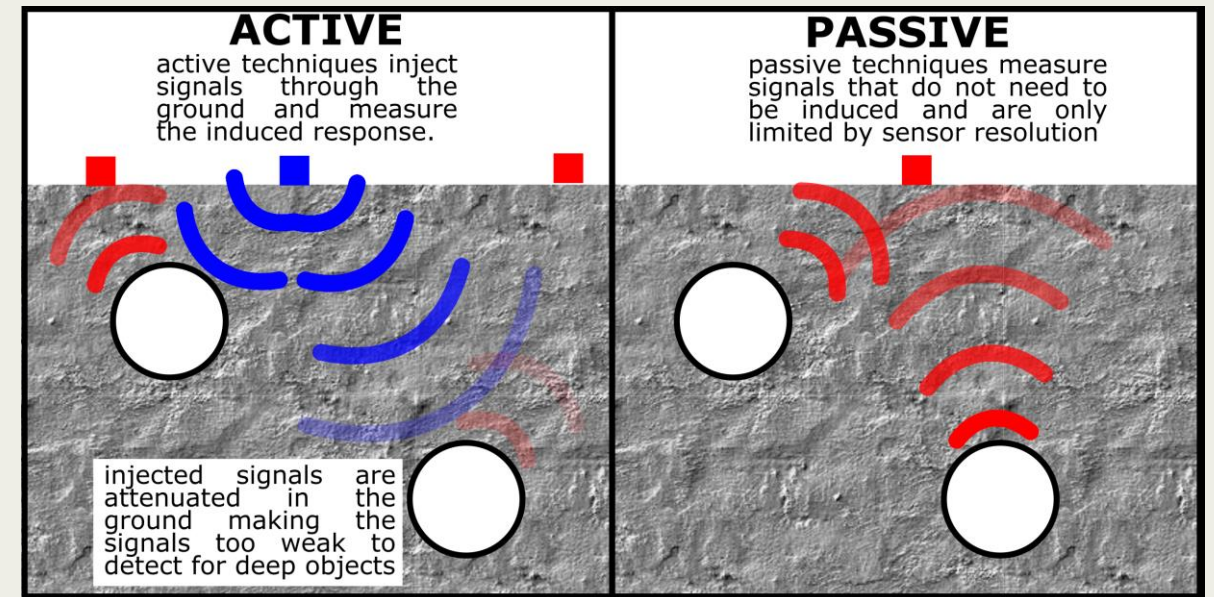


Introduction

Gravitational field mapping is used during an OSI to detect features with a density contrast which may indicate an underground nuclear explosion. Some of these are shown below.



Due to the passive nature of the measurement, the signal is not attenuated or shielded by ground conditions like other geophysical techniques, and has good resolution with depth. However, due to sensitivity to vibrational noise and the need to average this out, the technique is relatively slow to acquire data, leading to high time cost, low spatial resolution, and therefore a high inspection team functionality (ITF) risk.



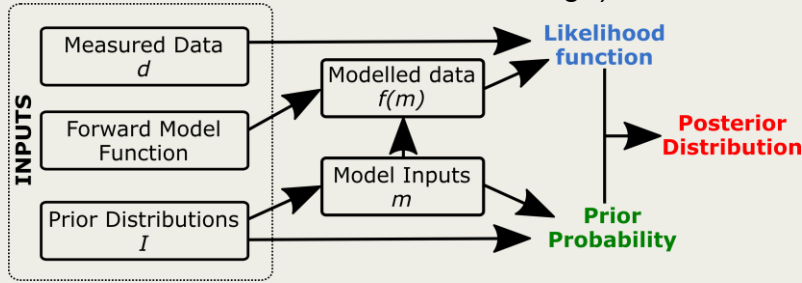
Furthermore, due to non-uniqueness of the measured gravity where multiple density distributions can give the same response at the surface, we use mathematical inversion techniques to iteratively link the data collected with a causative forward model of what is underground by comparing model outputs to the data.

For details of a field trial validation of the technique in civil engineering, see Boddice, D. et al. (2025) "An adaptive gravity and gravity gradient surveying strategy based on Bayesian inference", J. of Applied Geophysics X, XX-XX

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Bayesian Inference

One inversion approach to this is to use Bayesian inference to generate the posterior distribution which represents the probability distribution of the vector of model random variables (m) given the data vector (d) and the prior information (I) in the form of distributions for random variables (either informative normal distributions around expected values or non-informative uniform distributions over the whole range).



$$P(m|d, I) \propto P(d|m, I)P(m|I)$$

$$P(d|m, I) = \frac{1}{\sqrt{(2\pi)N_d|\Sigma|}} \exp\left[-\frac{1}{2}(d - f(m))^T \Sigma^{-1}(d - f(m))\right]$$

$$\Sigma = I_n \sigma_d$$

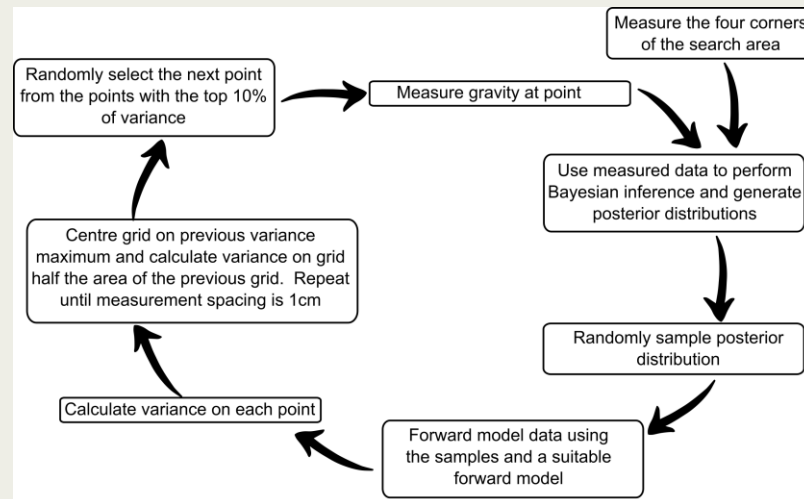
This is achieved using large number of models using a Monte Carlo approach and assessing the prior probability and likelihood using the formula above. A No U-turn sampler (NUTS) improves efficiency of choosing high likelihood models by ensuring rapid convergence of chains to high likelihood solutions. The technique is model agnostic, working with any geometric model which can be parameterised, as well as being suitable for any gravity or gravity gradient component.

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Looped Inference

Typically, inversion is conducted after a survey is completed requiring a full grid to be collected, requiring the operator to commit to a number of points and measurement spacing for the grid in advance. Looped inference uses the same inversion process but is conducted during the survey after every point is taken to determine the location of the next point.

This results from each inversion are used to prioritise sampling in spatial regions containing the most useful information to resolve ambiguity on the target feature parameters. By randomly sampling the generated posterior distribution to create multiple forward models, areas where there is large variability between models indicate the most productive, information rich areas to collect data to locate the buried feature. The process for carrying out looped inference is shown below



Test Methodology

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To test the method for OSI, regular grids and the proposed Bayesian Update (BU) approach were compared using synthetic data. A 500 x 500 m area with a horizontal emplacement tunnel of unknown size and location was used as a test scenario. Models used a cylinder forward model, characterised by centre X and Y coordinates, depth below ground, radius, length and rotation of the cylinder and density contrast with the ground. "True" parameters for the test are shown in the table below. Synthetic data was modelled for each grid and BU iteration along with normally distributed noise equivalent to the current CTBTO gravimeter (mean = 0, SD = 1.02). Priors were set to non-informative uniform distributions covering possible values.

Feature	Centre X	Centre Y	Depth	Radius	Rotation	Length	Density
Tunnel	101	25	20m	1.5 m	35°	200 m	-2800 kg/m ³

Inversion using 20000 iterations generated posterior distributions by fitting probability density functions. To assess performance:

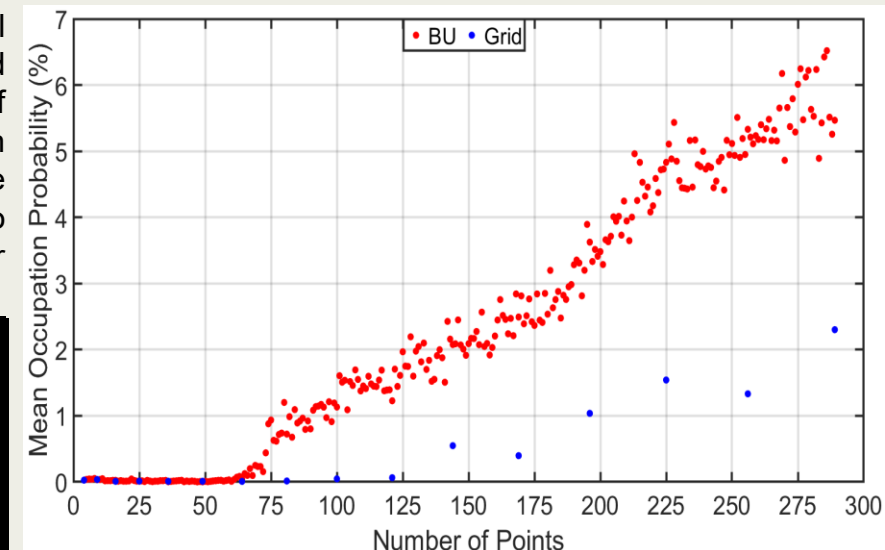
- Posterior distributions were calculated and compared to the true values of the feature to assess both mean and spread of proposed solutions
- A voxel grid of the underground space was used to calculate the percentage chance of a voxel containing the feature from the models. The mean occupation probability (MOP) of voxels contained within the true feature location was used as a metric to assess performance.



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Results

The figure below shows the positions of the points, tunnel location, and the posterior distributions for the model parameters, comparing grids with different spacing and an equivalent number of points collected using the looped BU algorithm. The mean occupation probability (MOP) contained within the tunnel is also shown to the right. Use of the BU algorithm starts to cluster the measurement points around the tunnel, thus providing higher value information to subsequent inversions. This allows the approach to reach a solution approximating the correct location of the tunnel (shown by higher MOP and posteriors clustered around true values) in less points (81 points as opposed to 144 points). The BU approach also reaches a more confident solution than the regular grid (shown by the smaller spread around the true values in the posterior) as the number of points increases.



Conclusions

The approach for the tested scenario showed an improvement in speed and accuracy for the detection of a tunnel compared to a regular grid approach.

- 43.75% less gravity points were required to locate the feature compared to a regular grid
- The maximum MOP was around 4% higher than the densest grid for the same number of points
- A similar MOP was obtained as the densest grid (289 points) using 150 points with the BU – 48 % less
- The approach can save time and reduce the ITF risk of a gravity mission.

