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Uncertainties in the SAUNA measurements - calibration revisited

A. Göök, P. Jansson, M. Aldener, T. Fritioff, A. Ringbom

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INTRODUCTION AND MAIN RESULTS

- Uncertainties in detection efficiencies and interference correction ratios estimated by earlier procedures were too large.
- Calibration procedure
  $\Rightarrow$ Covariance between the calibration parameters.
- A more robust curve fitting and enhanced uncertainty estimates were implemented.
- Resulting calibration parameters generally have smaller uncertainties.
- Impact of covariance on the uncertainties were studied.

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Overestimated uncertainties

Large uncertainties are delivered by the efficiency calibration program Xeff2. Example from a calibration measurement on a SAUNA III:

#b-gEfficiency			
XE-135	2	0.560483	0.129624
XE-133	3	0.664781	0.164466
XE-133	4	0.618714	0.098366
XE-131m	5	0.633318	0.084492
XE-133m	6	0.564280	0.075282
XE-133	7	0.220611	0.035146
XE-133	8	0.036837	0.005913
XE-133	9	0.164964	0.026294
XE-133	10	0.439025	0.069888

Large uncertainties are delivered by the efficiency calibration program Xeff2. Used scaling of uncertainties overestimates uncertainties in fit parameters. Calibration procedure involves extracting input parameters that are correlated.

$$\boxed{\varepsilon_\gamma^{30}} = \frac{A_{\beta\gamma}^{129}}{A_\beta^{CE} R_K}$$

$$\varepsilon_\gamma^{81} = \frac{B_\gamma^{81}}{B_\gamma^{30} - B_{\beta\gamma}^{129}} \frac{T_\gamma^{30}}{T_\gamma^{81}} \boxed{\varepsilon_\gamma^{30}}$$

$$\varepsilon_\gamma^{250} = R_{250/30}^{MC} \cdot \boxed{\varepsilon_\gamma^{30}}$$

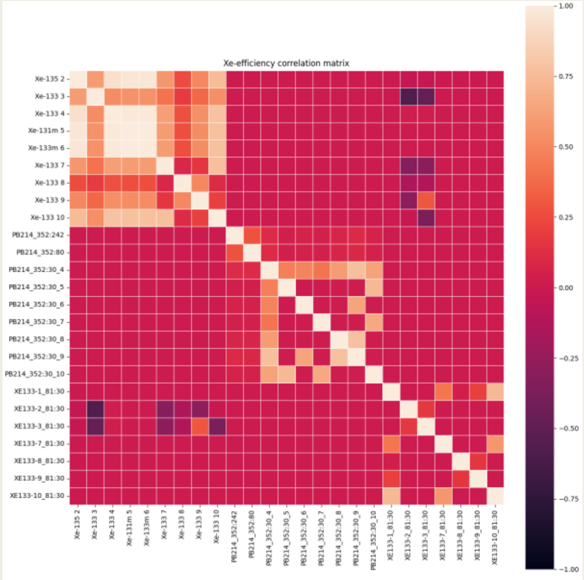
The ROI efficiencies are strongly correlated. Correlation between input parameters must be accounted for.

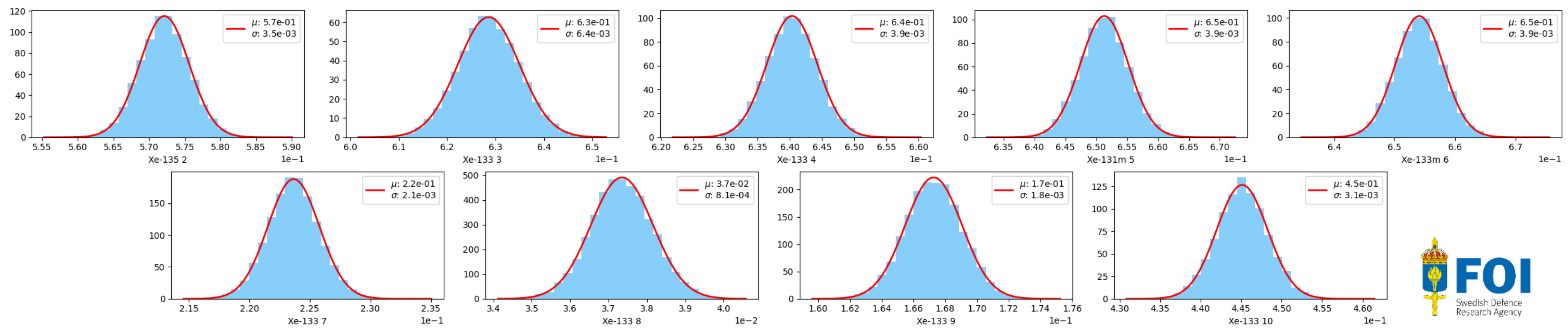
Monte-Carlo uncertainty propagation

What are the real uncertainties in the efficiency calibration?
Uncertainties in the efficiency calibration are statistical in nature. Have they been propagated correctly?
Uncertainties in the efficiency calibration assessed by resampling the calibration measurement data:

1. Each measurement is resampled by assuming $N_{ij}^{\beta\gamma} \sim Pois\left(N_{ij}^{\beta\gamma}\right)$
2. Repeat full calibration procedure for each sampled measurement to obtain samples of efficiency.
3. Estimate covariance matrix from the samples.

The Correlation Matrix





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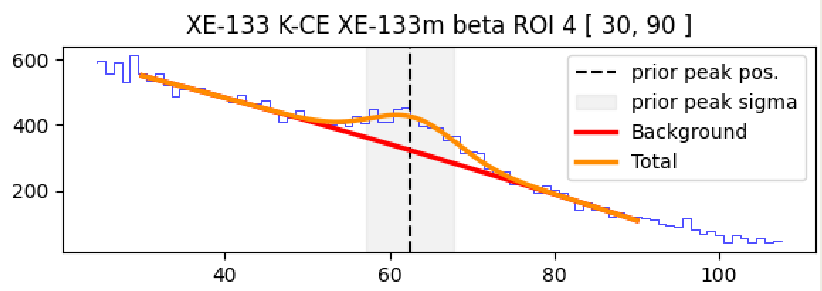
Peak Fitting with a Bayesian Prior

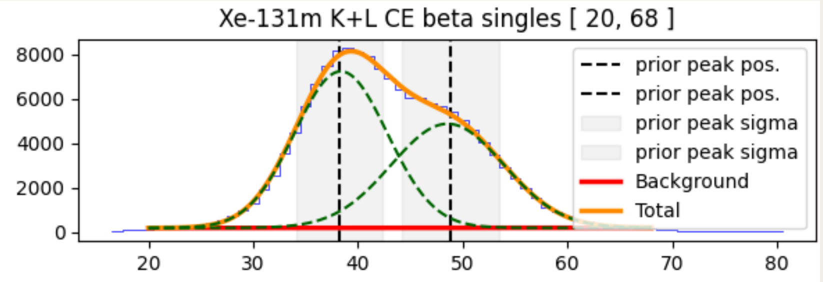
Peak fitting convergence could be challenging in Xeff2. Human intervention and experience to set fit limits needed for convergence.

- Small peak on large background.
- Close lying peaks.

Position and width known from energy and shape peak calibration.

- Use Bayesian prior.





Uncertainty propagation

Approximate the efficiency calculation as

$$\boldsymbol{f}(\boldsymbol{x}) \approx \boldsymbol{f}(\boldsymbol{x_0}) + \boldsymbol{J_f}(\boldsymbol{x} - \boldsymbol{x_0})$$

The efficiency covariance matrix is then

$$\boldsymbol{\Sigma_f} = \boldsymbol{J_f \Sigma_x J_f^T}$$

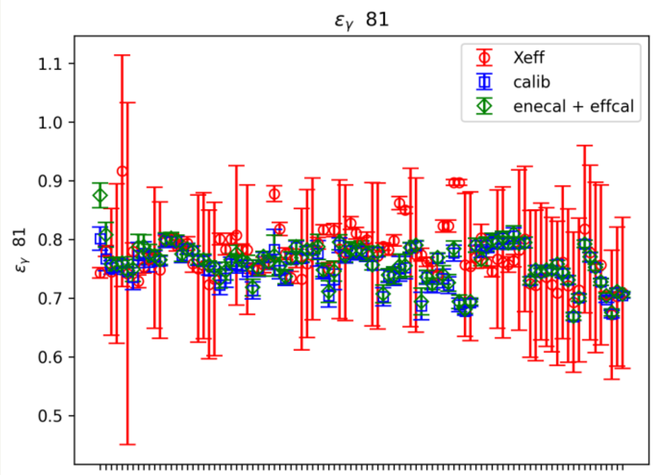
Analytical calculation of the Jacobian is cumbersome and prone to human error. Here implemented instead using `scipy.optimize.approx_fprime`.

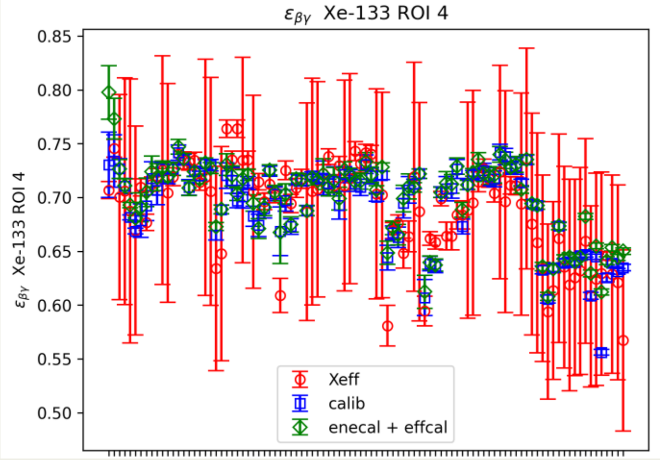
The input covariance must still be derived. We are using covariance from:

1. Parameters extracted from one and the same peak fit.
   Ex: A, μ, σ from one peak in a spectrum
2. Counts from (partially) overlapping ROIs.
   Ex: Counts in ROI 4 and ROI 5 & 6

Uncertainties correspond nicely to the Monte Carlo propagation results.

Comparison examples





Outlook

Work is on-going to refactor energy calibration to use Bayesian curve fitting. Covariance matrix of calibration parameters to be propagated into procedures to estimate activity concentrations (BGM).