SnT 2025

CTBT: SCIENCE AND TECHNOLOGY CONFERENCE

HOFBURG PALACE - Vienna and Online

8 TO 12 SEPTEMBER

Yield and Depth of NTS/DPRK Explosions Inverted from Regional Seismograms Using a New Algorithm

Chandan K. Saikia[1], Andrea Chiang[2] and Rongmao Zhou[1]

AFTAC, Patrick SFB, Florida 32925, USA[1]

Lawrence Livermore National Laboratory, Livermore, California 94551, USA[2]

INTRODUCTION

We develop a new algorithm to invert regional seismograms to extract yield and depth of explosions. Validated the method using synthetic seismograms of the NTS explosions generated by source parameters in Pasyanos and Chiang (2022). We compare the scalar moments estimated for the DC, CLVD and EX sources by this inversion method against those published results. The method is extended to yield and depth determination . .

METHODS/DATA

Current moment tensor estimation methods use Herrmann and Hutchenson (1993) in which the CLVD and EX GF contributions added to the DC GFs. It uses the same source strengths for the three sources. In this study, we keep these sources separate with different strengths and propagate wavefield to a station prior to summing them. Instead of inverting for the moment elements, we inverted for the scalar moment partition directly. Explosion seismograms are then extracted and modeled for the source parameters..

START

RESULTS

Surrogate synthetics for all NTS were inverted using the linear formulation that is built on the dependence on the 5 moment-tensor elements for the DC and the strengths of the explosion and CLVD sources. We find that while the estimate of scalar moments for the isotropic part is same, the estimates for the DC and CLVD are different, resulting in the different the partition percentages.

CONCLUSION

- Both method yields the same estimates for the isotropic scalar moment.
- Long-period GFs don't vary significantly as a function of depth over the range of the explosion depth.
- Amplitude and frequency content of the explosion source function vary quite a lot.
- We present new formulations to illustrate the partial derivatives of the TDSF as a function of yield and depth

Please do not use this spac, a QR code will be automatically overlayed

P2.1-391

Motivation

- Formulae in Minson & Dreger (2008) for inverting seismograms are based on the original formulae by Herrmann and Hutchenson (1993, will be referred to as HH) and have the following constraints on contributions from **EX (explosion), CLVD (Compensated Linear Vector Dipole) and DC (double-couple) sources**
    - (i) **EX, DC** and **CLVD** sources occur at the same depth.
    - (ii) Each source starts with the same diagonal moment tensor **(MT)** elements
    - (iii) Diagonal **GFs** are same for both **CLVD** and **EX** sources
    - (iv) Seismic waves propagate to the receivers as an effective source from the detonation point
- Constraint (ii) & (iii) allow GFs of the three source types to add up linearly for the diagonal moment elements.
- Essentially HH formulation is equivalent to adding the $\left[\frac{ZDD}{3}M_{xx}, \frac{ZDD}{3}M_{yy}, \frac{ZDD}{3}M_{zz}\right]$ term representing the CLVD and $\left[\frac{ZEX}{3}M_{xx}, \frac{ZEX}{3}M_{yy}, \frac{ZEX}{3}M_{zz}\right]$ term representing the EX-source to the MT formulation for the DC source (Langston,1981; Saikia and Herrmann, 1986).
- Above constraints are not necessarily true. **DC** source may be at a depth different from the other two. **EX** and **CLVD** sources are likely to confine at a shallow depth. The three sources may not have the same strength at the source.
- This study assumes the sources to act independently. Waves propagate to get added at a station.
- In this study, we established an algorithm that inverts waveforms to estimate the scalar seismic moments $M_{DC}, M_{CLVD},$ and M_{EX} of the three participating sources. Waveforms are expressed as a linear combination of a DC source comprising of 5-degree of freedom (e.g., 5 MT elements) M_{DC}^{j}, M_{CLVD} and M_{EX} sources; thus, invoking a total of 7 parameters. Following the inversion, scalar moment of the DC source is estimated using $M_{DC} = \sqrt{\sum_{j=1}^{6}\left[M_{DC}^{j}\right]^{2}/2}$.

Please do not use this space, a QR code will be automatically overlayed

Theoretical Basis for the Inversion Algorithm

THEORY

Langston (1981) and Saikia and Herrmann (1986) expressed the vertical wavefield for a double-couple system source in a cylindrical coordinate system as follows

$$U_z(t) = \left[\frac{ZSS(t)}{2}.\cos(2Az) - \frac{ZDD(t)}{2}\right]M_{xx} + \left[-\frac{ZSS(t)}{2}.\cos(2Az) - \frac{ZDD(t)}{2}\right]M_{yy} + [ZSS(t).\sin(2Az)]M_{xy} + [ZDS(t).\cos(Az)]M_{xz} + [ZDS(t).\sin(Az)]M_{yz}$$

By adding the terms $\left[\frac{ZDD(t)}{3}M_{xx} + \frac{ZDD(t)}{3}M_{yy} + \frac{ZDD(t)}{3}M_{zz}\right]$ for the CLVD and $\left[\frac{ZEX(t)}{3}M_{xx} + \frac{ZEX(t)}{3}M_{yy} + \frac{ZEX}{3}M_{zz}\right]$ for the EX-sources to above equation, one can write the following the expression

$$U_z = \left[\frac{ZSS(t)}{2}.\cos(2Az) - \frac{ZDD(t)}{6} + \frac{ZEX(t)}{3}\right]M_{xx} + \left[-\frac{ZSS(t)}{2}.\cos(2Az) - \frac{ZDD(t)}{6} + \frac{ZEX(t)}{3}\right]M_{yy} + \left[\frac{ZDD(t)}{3} + \frac{ZEX(t)}{3}\right]M_{zz}$$
$$+[ZSS(t).sin(2Az)]M_{xy} + [ZDS(t).\cos(2Az)]M_{xz} + [ZDS(t).\sin(Az)]M_{yz}$$

which is same as the expression in HH and has been widely used in all waveform inversion codes involving the F-K GFs.

- GFs are computed using the F-K integration technique (Saikia 1994; Zhu and Rivera, 2002). **The ZDD GFs for the CLVD is same as the ZDD GFs computed for the DC source.**
- The CLVD wavefield can be approximated by contrbution either from a force dipole or a single-couple mechanism. Saikia (2008, 2025) showed the amplitude of a single couple is **small by a factor of 2** compared to the double couple.

In this new algorithm, we allow DC, CLVD and EX sources to propagate their wavefields separately to a receiver station prior to the summing, which permits setting up of a linear inversion scheme for the scalar moment partition without having to invoke a MT matrix decomposition.

Background Mathematics for Matrix Formulation

$$[U] = [A][X]$$

$$|A| = \begin{vmatrix}
\frac{ZSS(t_1)}{2}\cos(2Az) - \frac{ZDD(t_1)}{2} & -\frac{ZSS(t_1)}{2}\cos(2Az) - \frac{ZDD(t_1)}{2} & ZSS(t_1)\sin(2Az) & ZDS(t_1)\cos(Az) & ZDS(t_1)\sin(Az) & \overline{ZDD(t_1)} & ZEX(t_1) \\
\frac{RSS(t_1)}{2}\cos(2Az) - \frac{RDD(t_2)}{2} & -\frac{RSS(t_1)}{2}\cos(2Az) - \frac{RDD(t_2)}{2} & RSS(t_1)\sin(2Az) & RDS(t_1)\cos(Az) & RDS(t_1)\sin(Az) & \overline{RDD(t_1)} & REX(t_1) \\
\frac{TSS(t_1)}{2}\sin(2Az) & -\frac{TSS((t_1)}{2}\sin(2Az) & -TSS(t_1)\cos(2Az) & TDS(t_1)\sin(Az) & -TDS(t_1)\cos(Az) & 0 & 0 \\
\frac{ZSS(t_2)}{2}\cos(2Az) - \frac{ZDD(t_2)}{2} & -\frac{ZSS(t_2)}{2}\cos(2Az) - \frac{ZDD(t_2)}{2} & ZSS(t_2)\sin(2Az) & ZDS(t_2)\cos(Az) & ZDS(t_2)\sin(Az) & \overline{ZDD(t_2)} & ZEX(t_2) \\
\frac{RSS(t_2)}{2}\cos(2Az) - \frac{RDD(t_2)}{2} & -\frac{RSS(t_2)}{2}\cos(2Az) - \frac{RDD(t_2)}{2} & RSS(t_2)\sin(2Az) & RDS(t_2)\cos(Az) & RDS(t_2)\sin(Az) & \overline{RDD(t_2)} & REX(t_2) \\
\frac{TSS(t_2)}{2}\sin(2Az) & -\frac{TSS(t_2)}{2}\sin(2Az) & -TSS(t_2)\cos(2Az) & TDS(t_2)\sin(Az) & -TDS(t_2)\cos(Az) & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{ZSS(t_m)}{2}\cos(2Az) - \frac{ZDD(t_m)}{2} & -\frac{ZSS(t_m)}{2}\cos(2Az) - \frac{ZDD(t_m)}{2} & ZSS(t_m)\sin(2Az & ZDS(t_m)\cos(Az) & ZDS(t_m)\sin(Az) & \overline{ZDD(t_m)} & ZEX(t_m) \\
\frac{RSS(t_m)}{2}\cos(2Az) - \frac{RDD(t_m)}{2} & -\frac{RSS(t_m)}{2}\cos(2Az) - \frac{RDD(t_m)}{2} & RSS(t_m)\sin(2Az) & RDS(t_m)\cos(Az) & RDS(t_m)\sin(Az) & \overline{RDD(t_m)} & REX(t_m) \\
\frac{TSS(t_m)}{2}\sin(2Az) & -\frac{TSS(t_m)}{2}\sin(2Az) & -TSS(t_m)\cos(2Az) & TDS(t_m)\sin(Az) & -TDS(t_m)\cos(Az) & 0 & 0
\end{vmatrix}$$

$$[U] = [U_z(t_1) \quad U_R(t_1) \quad U_T(t_1) \quad U_z(t_2) \quad U_R(t_2) \quad U_T(t_2) \quad \cdots \quad \cdots \quad U_z(t_m) \quad U_R(t_m) \quad U_T(t_m)]^{T}$$

$$[x] = [M_{xx}^{DC} \quad M_{yy}^{DC} \quad M_{xy}^{DC} \quad M_{xz}^{DC} \quad M_{yz}^{DC} \quad M_{CLVD} \quad M_{EX} \quad]^{T}$$

$$[X] = [A^T A]^{-1} A^T [U]$$

VALIDATION OF THE INVERSION SCHEME

- Using the mathematical steps discussed in the previous slides, codes to use both HH and the current formulations were developed.

- For the current theory, we constructed data seismograms for a double-couple mechanism with dip, slip and strike of 30^o, 40^o and 165^o, respectively, a vertically symmetric CLVD and an explosion source, separately. The seismograms were summed after scaling each by a scalar seismic moment partition for the three seismic sources: DC=0.30, CLVD=0.15, and EX=0.55.

- For HH, we used the MT elements published top construct seismograms for five NTS explosions, namely Hornitos (1989/10/31), Barnwell (1989/12/08), Houston (1990/11/14), Hoya (1991/11/14) and Junction (1992/03/26) (Pasyanos and Chiang, 2022). GFs were computed using the WUS velocity model (Herrmann et al., 2011).

- For each event, we used 8 or 9 station network for the code validation.

- Study was conducted for different partition values for the DC, CLVD and EX sources. In each case, the solutions were exactly recovered.

- After the validation, we applied the method to all NTS and Korean Nuclear explosions.

SnT 2025
CTBT: SCIENCE AND TECHNOLOGY CONFERENCE
HOFBURG PALACE - Vienna and Online
8 TO 12 SEPTEMBER

Application of the New Algorithm to Surrogate NTS Synthetic Seismograms

- Pasyanos and Chiang (2022; will be referred to as PC) inverted regional waveforms of 133 nuclear and 9 chemical explosions from the phase I & II source-physics experiment and the Non-Proliferation Chemical Explosion (NPE) of Sept 22, 1993. They summarized their results in Table 1 of the paper and in the supplements (can be downloaded from the BSSA archive). They used the WUS velocity model to compute GFs.

- Built a library of GFs to implement the proposed algorithm. Used the WUS model. Computed synthetic seismograms to treat as the recorded data. Co-author Dr. Chiang inverted the observed waveforms for the published MT solutions published in PC with a high-quality agreement to the data.

- Advantage of Using a Library of synthetic seismograms

  no alignment of waveforms
  no cycle skipping

RESULTS

- recovered solution exactly when synthetic seismograms were computed using the HH formulation
- isotropic scalar moments are same for both algorithms
- scalar moments of the CLVD and DC sources were different
- HH required decomposition of the moment tensor matrix
- Current algorithm establishes scalar moment partition directly

INTRODUCTION
OBJECTIVES
METHODS/DATA
RESULTS
CONCLUSION

Yield and Depth Estimation
Source Time Function

$$S(w,h,t) = \underbrace{\left[\frac{R}{4\mu}\right]\left[\frac{C^2}{\beta p}\right]\left[\left[\partial_t[H(t)e^{-\alpha t}\sin(pt)]\right]\right]}_{A(t,hW)} * \underbrace{\left[e^{-\gamma t}P_1 + P_2\right]}_{B(t,h,W)} = A(t,h,W) + B(t,h,W)$$

See Saikia (2017) for the variable description. C is the compressional velocity and $\beta = \sqrt{(\lambda + 2\mu)/4\mu}$, and other variables are as follows and taken from Saikia (2025)

$$P_1 = P_{10}\left[\frac{h}{h_o}\right] \quad P_2 = P_{20}\left[\frac{h_o}{h}\right]^{\frac{n-9}{3n}} W^{-0.13} \quad \gamma = \gamma_o\left[\frac{h}{h_o}\right]^{\frac{1}{n}} W^{-\frac{1}{3}} \quad \propto = \frac{\omega_o}{2\beta} \quad p = \omega_o\sqrt{\frac{1}{\beta} - \frac{1}{4\beta^2}} \quad \omega_o = \frac{C}{R_o}\left[\frac{h}{h_o}\right]^{\frac{1}{n}} W^{-\frac{1}{3}}$$

$$U_{EX}(t) = A(t,h,W) * B(t,h,W) * GF_{EX}(t,h)$$

In poster P2.2-389, we illustrated that long-period F-K $G_{EX}(t,h)$ green's functions at shallow depth (shallower than 1.5 Km) are not sensitive to the depth variation. Note that, $U_{EX}(t)$ in above expression is dependent non-linearly on (W,h). Hence, we need to set up a non-linear iterative least-squares inversion for W and h. We expand $U_{EX}(t,h,W)$ about an initial starting values of W,h such that

$$U_{EX}(t, h + \triangle h, W + \triangle W) = U_{EX}(t,h,W) + \frac{\partial U_{EX}}{\partial h}\triangle h + \frac{\partial U_{EX}}{\partial W}\triangle W + \ \ldots$$

$$|\Delta U_{EX}| = \begin{vmatrix} \partial_h U_{EX}(t_1,h,W) & \partial_W U_{EX}(t_1,h,W) \\ \vdots & \vdots \\ \partial_h U_{EX}(t_m,h,W) & \partial_W U_{EX}(t_m,h,W) \end{vmatrix} \begin{vmatrix} \Delta h \\ \Delta W \end{vmatrix} \qquad [\Delta U_{EX}] = [A][\Delta X] \qquad [\Delta X] = [A^T A]^{-1} A^T [\Delta U_{EX}]$$

YYIELD & DEPTH ESTIMATION ……

Yield and depth derivatives of the explosion wavefield can be expressed

$$\partial_h U_{EX} = \partial_h A(t,h,w) * B(t,h,w) * G_{EX}(t,h) + A(t,h,w) * \partial_h B * G_{EX}(t,h) \qquad \partial_h G_{EX} = 0$$

$$\partial_w U_{EX} = \partial_w A(t,h,w) * B(t,h,w) * G_{EX}(t,h) + A(t,h,w) * \partial_w B * G_{EX}(t,h)$$

where the partial derivatives of the source function with yield (*w*) and depth (*h*) are as follows

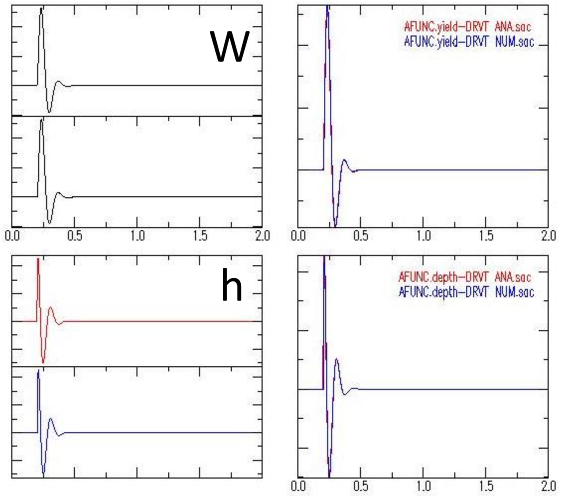
$$\partial_h B = -\frac{\gamma}{n}\frac{te^{-\gamma t}}{h}[P_1 - P_2] - \frac{e^{-\gamma t}}{h}P_1 + \frac{(n-9)[e^{-\gamma t}-1]}{3n}\frac{P_2}{h}$$

$$\partial_w B = \frac{\gamma t}{3w}e^{-\gamma t}[P_1 - P_2] - \frac{0.13}{w}(e^{-\gamma t}-1)P_2$$

$$\partial_h A = \frac{c}{4\mu\beta p}\left[\frac{1}{nh}\right]\partial_t[-\alpha t.e^{-\alpha t}\sin(pt) + pt.e^{-\alpha t}.\cos(pt) - p.e^{-\alpha t}\sin(pt)]$$

$$\partial_w A = \frac{c}{4\mu\beta p}\left[\frac{1}{3w}\right]\partial_t[e^{-\alpha t}\{(1+\alpha t)\sin(pt) - pt.\cos(pt)\}]$$

Figure to the right confirms the accuracy of the partial derivatives presented to the left where we computed the yield and depth derivatives for parameters given below and compared them against the results from the numerical computation.



- ∂_w is computed for a w of 500 Kt and its numerical derivative using ΔW=0.1Kt for the explosion at 800m depth.
- ∂_h is computed at a depth of 800 and its numerical derivative using Δh=20m for w=500 Kt yield explosion.

Calculation of Moment Partitions

M
z
z

Parameters of NTS Explosions Used in this Study

Event	Date	Origin Time	Lat (N)	Lon(W)	h	W	M_o	M_{EX}	Mw/mb
Hornitos	1989/10/31	15:29:59.39	37.2633	116.4638	564	20-150	2.30	1.39	4.8/5.7
Barnwell	1989/12/08	14:59:59.28	37.2474	116.4145	601	20-150	1.88	1.07	4.8/5.6
Houston	1990/11/14	19:16:59.26	37.2262	116.3671	595	20-150	1.55	0.91	4.7/5.4
Hoya	1991/09/14	18:59:59.41	37.2334	116.4103	671	20-150	3.34	1.88	5.0/5.5
Junction	1192/03/26	16:29:59.52	37.2327	116.3135	640	20-150	1.80	1.09	4.8/5.6

h,W : depth in meter and yield in kT taken from Catalog of Worldwide Nuclear Testing, V. V. Mikhailov.

M_O, M_{EX} : scalar and explosion seismic moments in x10^{16} N-m(taken from Pasyanos & Chiang,2022)

Herrmann and Hutchenson (1993)

eigen Values of moment matrix M =$[\lambda_1, \lambda_2, \lambda_3]$

$$\boldsymbol{EX} = \frac{tr(M)}{3.\,[\max(\lambda_1, \lambda_2, \lambda_3)]}$$

$$\widetilde{M} = M - \frac{tr(M)}{3}$$: Deviatoric Matrix

eigen Values of $\widetilde{M} = [\lambda_1', \lambda_2', \lambda_3']$

$$\epsilon = -\frac{\min[abs(\lambda_1'), abs(\lambda_2'), abs(\lambda_3')]}{\max[abs(\lambda_1'), abs(\lambda_2'), abs(\lambda_3')]}$$

$$\boldsymbol{CL} = -2.\varepsilon.\,abs\big(1.0 - abs(EX)\big)$$

$$\boldsymbol{DC} = 1.0 - abs(CL) - abs(EX)$$

This Study

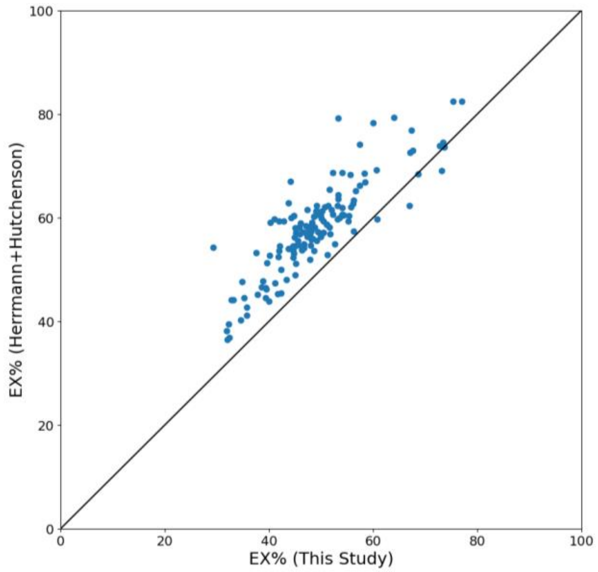
$$M_o = M_{DC} + M_{CLVD} + M_{EX}$$

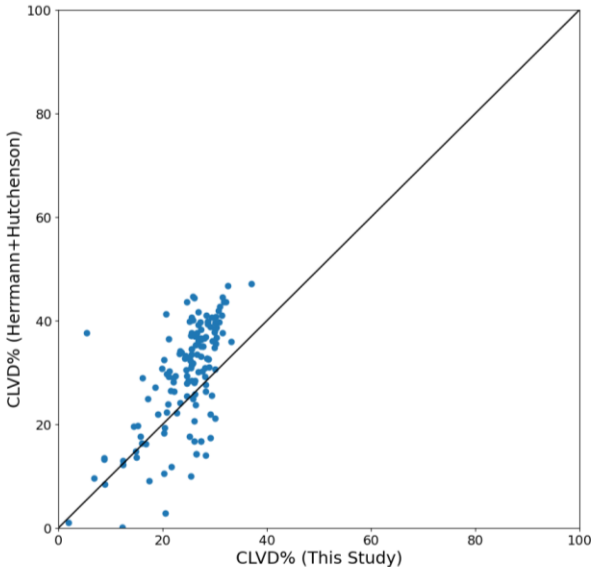
$$M_{DC} = f(M_{xx}, M_{yy}, M_{xy}, M_{xz}, M_{yz}, -(M_{xx} + M_{yy})$$

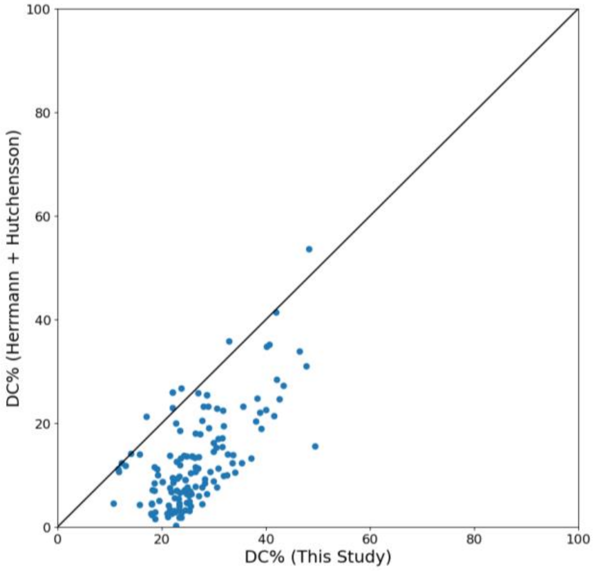
$$M_{DC} = \sqrt{\left\{\sum_{j=1}^{6} M_j^2\right\}/2}$$

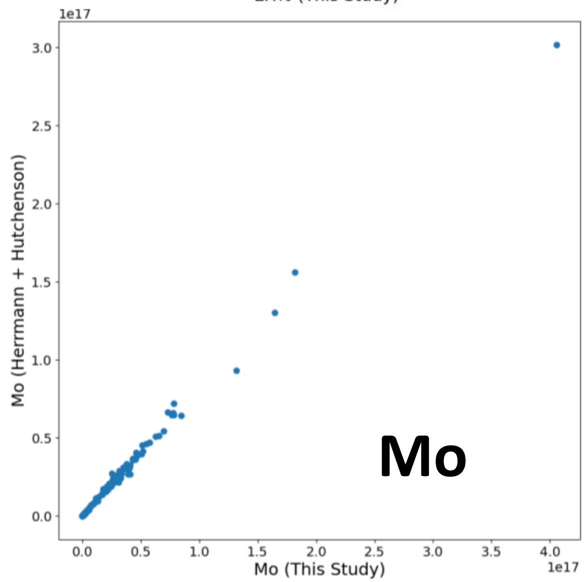
$$\boldsymbol{DC} = \frac{M_{DC}}{M_o} \qquad \boldsymbol{CL} = \frac{M_{CLVD}}{M_o} \qquad \boldsymbol{EX} = \frac{M_{EX}}{M_o}$$

RESULTS FOR NTS EXPLOSIONS





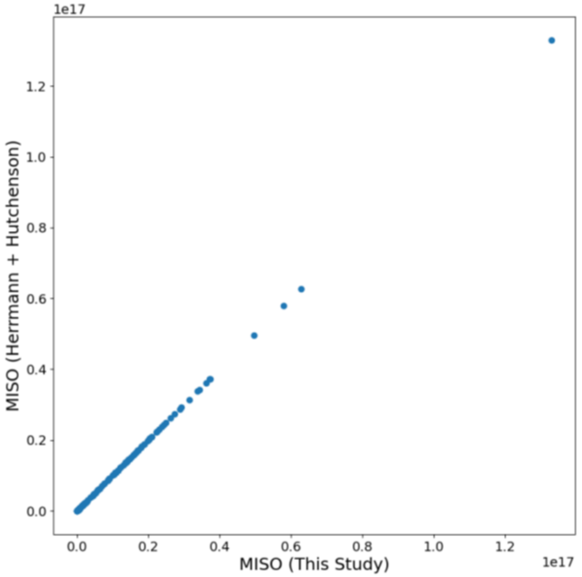




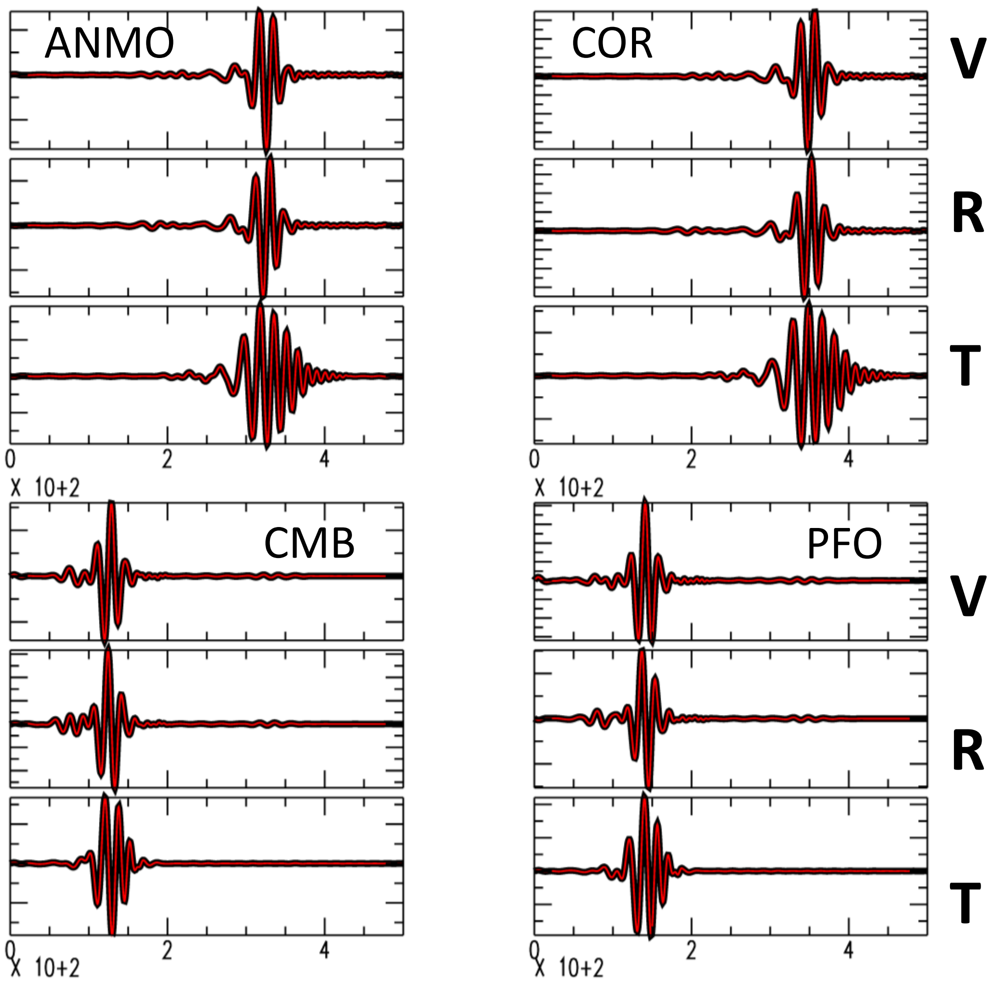
HH: MISO/[EX%]

TS: $$\sqrt{\left[\sum_{j=1}^{6} M_j^{DC}\right]/2 + M_{CLVD} + M_{EX}}$$

$$M_6^{DC} = -\left(M_1^{DC} + M_2^{DC}\right)$$



Waveform Comparison for a NTS Explosion Using Herrmann & Hutchenson Formula vs. this study



Taken from Pasyanos & Chiang (2022) and HH to construct data

M_{xx}= 1.188e+23
M_{yy}= 1.348e+23
M_{zz}= 3.113e+23
M_{xy}=-2.400e+23
M_{xz}=-4.630e+22
M_{yz}= 4.450e+22

Used the new formulation to construct the matrix [A] for inversion

M_{xx}=2.7297e+22
M_{xy}=4.3356e+22
M_{xz}=-2.4101e+22
M_{yy}=-4.6301E+22
M_{yz}==-4.4513e+22
M_{CLVD}=9.65104e+22
M_{EX}=1.8852e+23

CONCLUSIONS

- Formulated the mathematical background for an inversion algorithm to estimate yield and depth of an explosion when accompanied by DC and CLVD sources.
- The algorithm uses GFs for fundamental faults and explosions from a velocity model and inverts for the scalar moment partition to the DC, CLVD and EX sources.
- **The DC:CLVD:EX partitions estimated by the two methods are different**
- Scalar seismic moments of the ISO source estimated by the two methods agree well.
- Estimating the yield and depth of explosions using the partial derivatives of the wavefield with respect to *W* and *h* is on-going.