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Quantifying atmospheric uncertainty and detection quality impacts on infrasound localization

Infrasound Technology Workshop (ITW2024)

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Outline

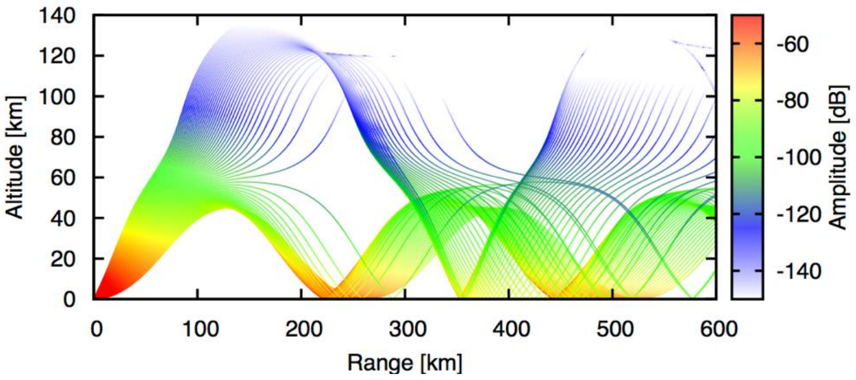
- Back Projection Localization
- Examples
  - Multi-phase single-station localization
  - Regional multi-station localization
- Localization Confidence Trends
- Summary and Ongoing R&D

Background – Auxiliary Parameters

- The equations defining rays in an inhomogeneous, moving atmosphere are:

$$\frac{\partial \vec{x}}{\partial s} = \frac{\vec{c}_g}{c_g}, \quad \frac{\partial \psi_j}{\partial s} = -\frac{1}{c_g}\left(\psi \frac{\partial c}{\partial x_j} + \vec{\psi} \cdot \frac{\partial \vec{v}_0}{\partial x_j}\right), \quad \vec{c}_g = c\frac{\vec{\psi}}{\psi} + \vec{v}_0$$

- Auxiliary parameters defining variations with respect to the initial inclination and azimuth angles can be introduced to solve the Transport equation, $\vec{X}^{(\vartheta)} = \frac{\partial \vec{x}}{\partial \vartheta}, \ \vec{\Psi}^{(\vartheta)} = \frac{\partial \vec{\psi}}{\partial \vartheta}$



- Additional differential equations are defined by taking derivatives of the above,

$$\frac{\partial \vec{X}^{(\vartheta)}}{\partial s} = \frac{\partial}{\partial \vartheta} \frac{\vec{c}_g}{c_g}$$

- Auxiliary parameters were originally investigated to solve the transport equation, define geometric losses, and identify caustics (Blom & Waxler, 2012)

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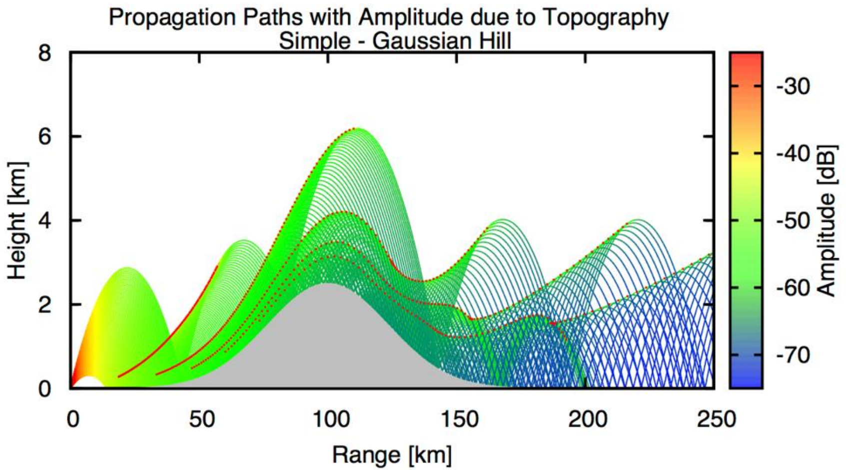

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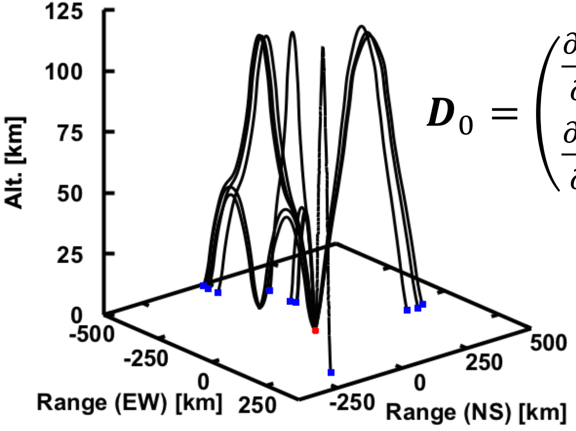
Background – Auxiliary Parameters

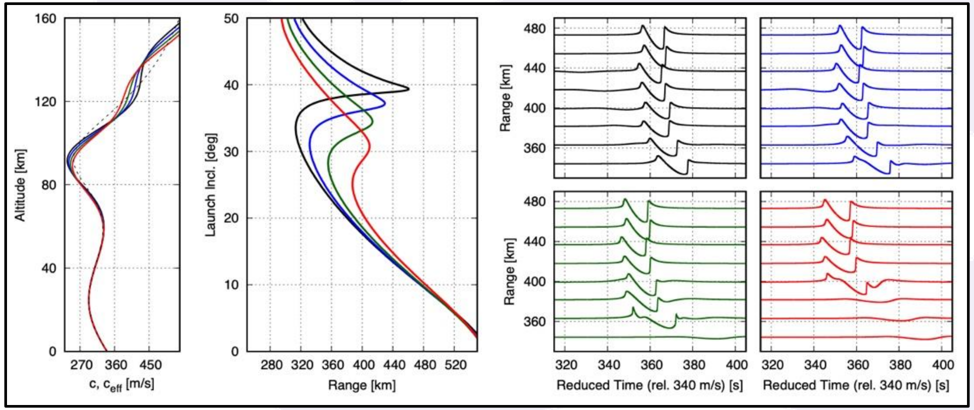
- Auxiliary parameters were subsequently leveraged as part of a Levenberg-Marquardt eigenray algorithm
- The Jacobian determinant is used in weakly non-linear waveform evolution computations

$$\Delta\vec{\Omega} = \left(\boldsymbol{D}_0 + \lambda\,\mathrm{diag}(\boldsymbol{D}_0)\right)^{-1}\Delta\vec{x}_0,$$



$$\boldsymbol{D}_0 = \begin{pmatrix} \frac{\partial x_0}{\partial \vartheta} & \frac{\partial x_0}{\partial \varphi} \\ \frac{\partial y_0}{\partial \vartheta} & \frac{\partial y_0}{\partial \varphi} \end{pmatrix}$$

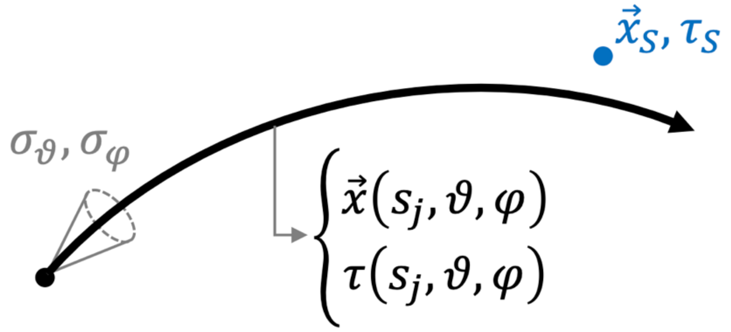
$$\frac{\partial u}{\partial s} = \tilde{\beta} u \frac{\partial u}{\partial \tau}, \quad \tilde{\beta} = \beta \frac{p_{\mathrm{ref}}}{\rho_0 c^2} \frac{\psi_0 c_0}{c_{g,0} c_{\mathrm{src}}} \sqrt{\frac{\rho_0 c_{g0}^3 D_0}{\rho c_g^3 D} \frac{c_g \psi^3}{c_{g0} \psi_0^3}}$$



Back Projection Localization

- During discussions with Garth Frazier at Ole Miss NCPA, the idea came up to use the auxiliary parameters in a back projection localization algorithm

- General idea:
  - Infrasound stations (arrays) detect an event providing coherence (SNR) and direction-of-arrival information (back azimuth and trace velocity)
  - Compute time-reversed ray path for each detected phase and use the auxiliary parameters to quantify the probability of a source at a hypothetical location and origin time, $\vec{x}_S$ and τ_S, producing that detection when the spatial point doesn't lie along the ray path.



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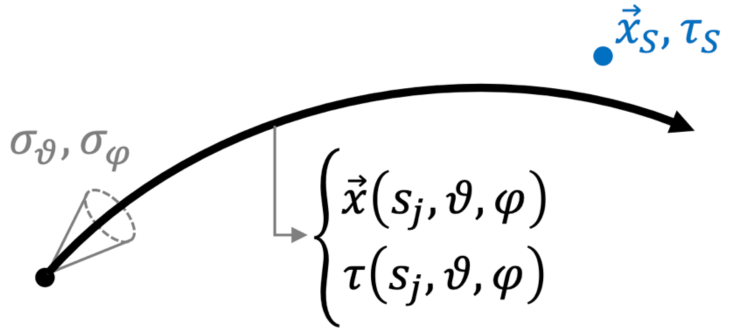
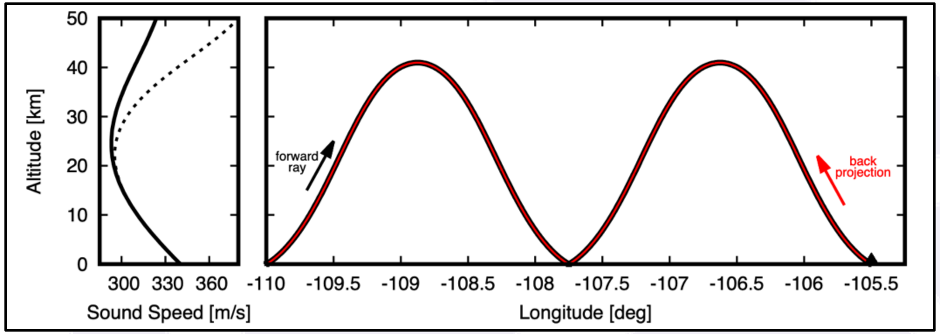


- Considerations:
  - Implemented auxiliary parameters are only spatial. Can we define variation of propagation time along the ray path? *(not included here, see manuscript)*
  - How can we quantify uncertainty/confidence in detection direction-of-arrival estimates?
  - Localization should include uncertainty in the atmospheric state, but avoid introducing many parameters (e,g., EOF coefficients)?

Back Projection Localization – Ray Tracing

- Time-reversed ray tracing requires swapping the wind flow direction (east ↔ west, north ↔ south)
- In spherical geometry, the inclination angles at launch and arrival are different, so reciprocity is a bit more complicated (but still valid)



	Eigenray	Time-Reversed Projection
Starting Location	30°, -110°	30°, -105.5°
Arrival Location	30°, -105.4998°	29.9999°, -110.0002°
Launch Inclination	9.32190589°	9.3168453°
Arrival Inclination	9.3168453°	9.32190589°
Launch Azimuth	88.893277°	-88.85578°
Arrival Back Azimuth	-88.85578°	88.893263°
Propagation Time	1484.417 sec	1484.425 sec
Transport Coefficient (rel. 1 km)	-43.1139 dB	-43.1138 dB

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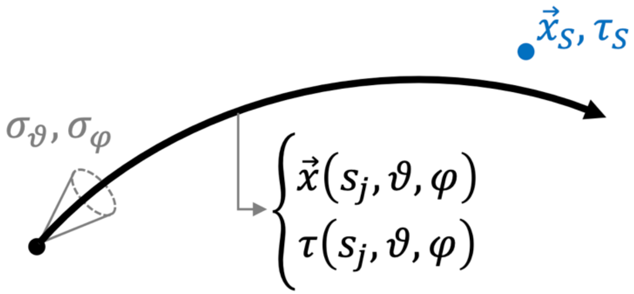
Back Projection Localization – Likelihood

- The likelihood defines the probability of observing a detection in the case that a given source hypothesis is true,

$$P(D_k|S,A)=\sum_j \frac{e^{-\frac{1}{2}\sum_\alpha\left(\frac{\alpha-\alpha_S}{\sigma_\alpha}\right)^2}}{4\pi\prod_\alpha \sigma_\alpha}, \qquad \alpha\left(s_j,\vartheta_k,\varphi_k|\,A\right)=x,y,z,\tau$$



- The ray path solution defines the various α values, and the auxiliary parameters relate variations in the launch angles to those in α, for example,

$$\sigma_x=\sqrt{\left(\frac{\partial x}{\partial\vartheta}\sigma_\vartheta\right)^2+\left(\frac{\partial x}{\partial\varphi}\sigma_\varphi\right)^2}=\sqrt{(X^{(\vartheta)}\sigma_\vartheta)^2+\left(X^{(\varphi)}\sigma_\varphi\right)^2}$$

…we just need to define σ_ϑ and σ_φ

Back Projection Localization – Uncertainty

- Previous likelihood construction defined a relation between an azimuthal probability distribution function (von Mises) and the Fisher statistic (SNR) of the detection:

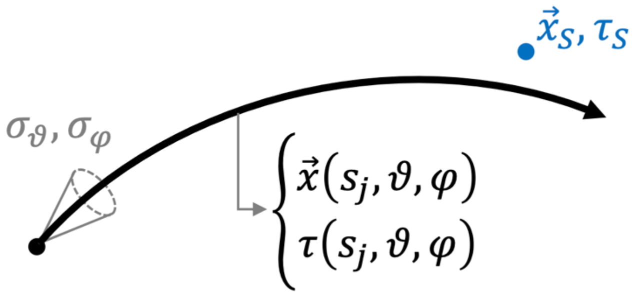
$$\rho_{\mathrm{az}}(D_k|\varphi_S) = \frac{e^{\kappa\ \cos(\varphi_S - \varphi_k)}}{2\pi I_0(\kappa)}, \qquad \kappa = 2(\mathrm{q}-1)\mathcal{F}, \qquad \mathrm{q} = \text{elements in array}$$

- Expanding for small differences, $\varphi_S - \varphi_k = \delta_\varphi$,

$$\rho_{\mathrm{az}}(D_k|\varphi_S) = \frac{e^{\kappa\left(1+\frac{1}{2}\delta_\varphi^2 + O(\delta_\varphi^4)\right)}}{2\pi I_0(\kappa)}$$

$$\rightarrow \sigma_\varphi = \frac{1}{\sqrt{\kappa}} = \frac{1}{\sqrt{2(\mathrm{q}-1)\mathcal{F}}}$$

  - Detection with $\mathrm{q} = 6, \mathcal{F} = 20$: $\sigma_\varphi = 4°$



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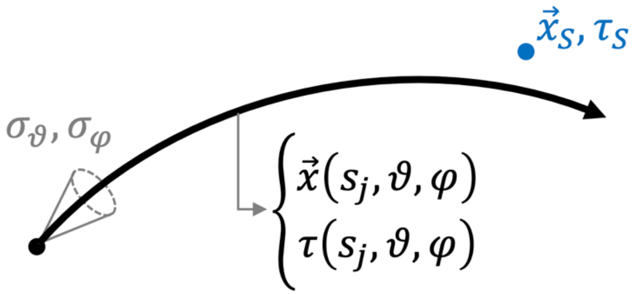
- Assuming the distribution is symmetric in slowness space, trace velocity variance is similarly defined:

$$\sigma_v = v_k \sigma_\varphi$$

  - Detection with $\mathrm{q} = 6,\ \mathcal{F} = 20,\ v_k = 360\ \frac{\mathrm{m}}{\mathrm{s}}$: $\sigma_v = 25\,\frac{\mathrm{m}}{\mathrm{s}}$



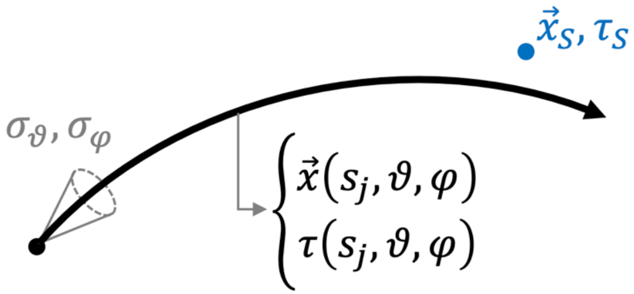
Back Projection Localization – Uncertainty

- Relating trace velocity to inclination angle requires knowledge of the ambient sound speed, so uncertainty in that quantity must also be considered

$$\vartheta = \cos^{-1}\frac{c_0}{v_k} \rightarrow \sigma_\vartheta = \sqrt{\left(\frac{\partial\vartheta}{\partial v_k}\sigma_v\right)^2 + \left(\frac{\partial\vartheta}{\partial c_0}\sigma_c\right)^2}$$

$$\sigma_\vartheta^2 = \frac{\sigma_c^2 + \left(\frac{c_0}{v_k}\right)^2\sigma_v^2}{v_k^2 - c_0^2}$$

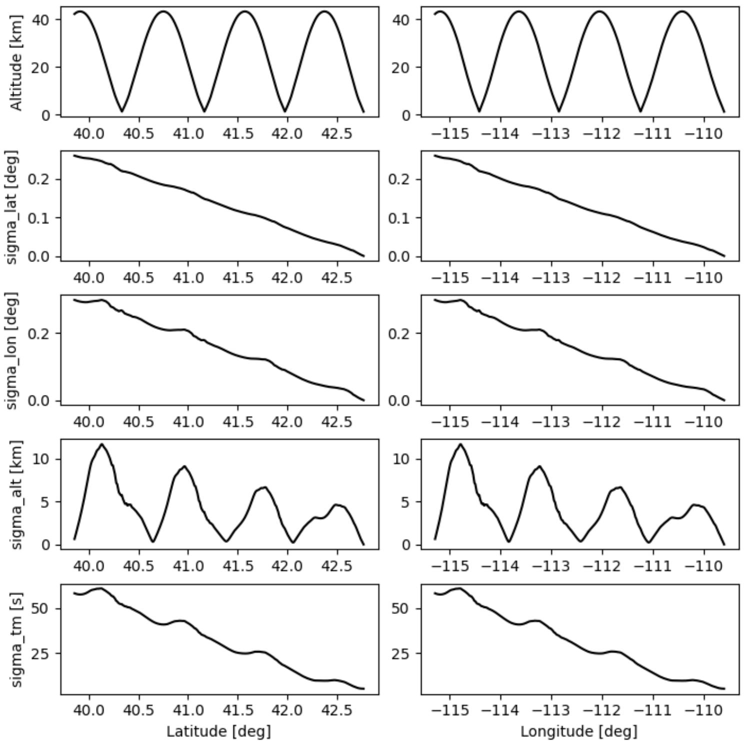
  - Detection with $\mathrm{q} = 6, \mathcal{F} = 20, v_k = 360\,\frac{\mathrm{m}}{\mathrm{s}}, c_0 = 340\frac{\mathrm{m}}{\mathrm{s}}, \sigma_c = 5\frac{\mathrm{m}}{\mathrm{s}}: \vartheta = 19.2°, \sigma_\vartheta = 11.9°$



- Some complications at shallow angles when the denominator grows large, but the physical constraint of $\vartheta \geq 0°$ can be used to limit σ_ϑ

Back Projection Localization – Uncertainty

- A synthetic event has been constructed to evaluate the back projection method
- Standard deviations along the time-reversed paths exhibit reasonable behavior



- Latitude and longitude variations increase at nearly constant rate with propagation range
  - note: propagation to the southwest, so increased ray length left to right in figure
  - At 40° latitude, $\sigma_\phi = 0.2°$ ~17 km

- Propagation time uncertainty is on the order of 10’s of seconds and increases like latitude/longitude

- Caustics near the turning heights show up in σ_z

Back Projection Localization – Atmo Uncertainty

- The formulation thus far uses a **single atmospheric specification**:

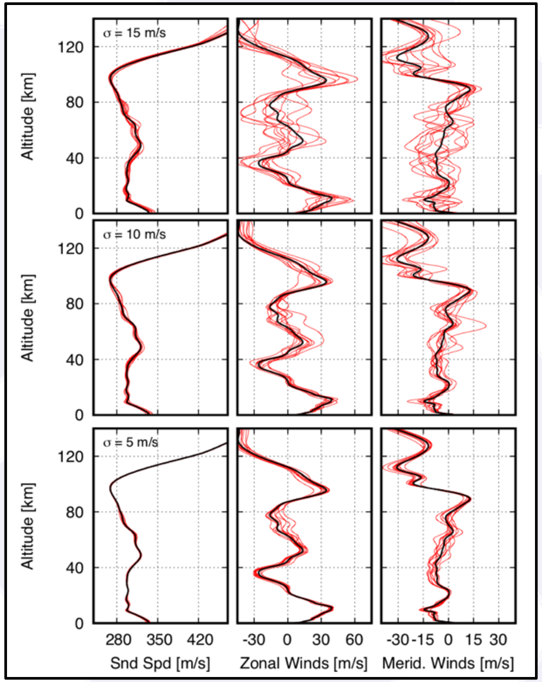
$$P(S|\{D_1, D_2, \ldots D_K\}, A) = \frac{\prod_k P(D_k|S, A)}{P(\{D_1, D_2, \ldots D_K\})} P_0(S)$$

- Quantify uncertainty by considering an **atmospheric ensemble**:

$$P(S|\{D_1, \ldots, D_K\}, \{A_1, \ldots, A_M\}) = \frac{\sum_m \prod_k P(D_k|S,Am) P_0(A_m)}{P(\{D_1,D_1 \ldots,D_K\})} P_0(S)$$

- Note: **atmospheric sounding** application:

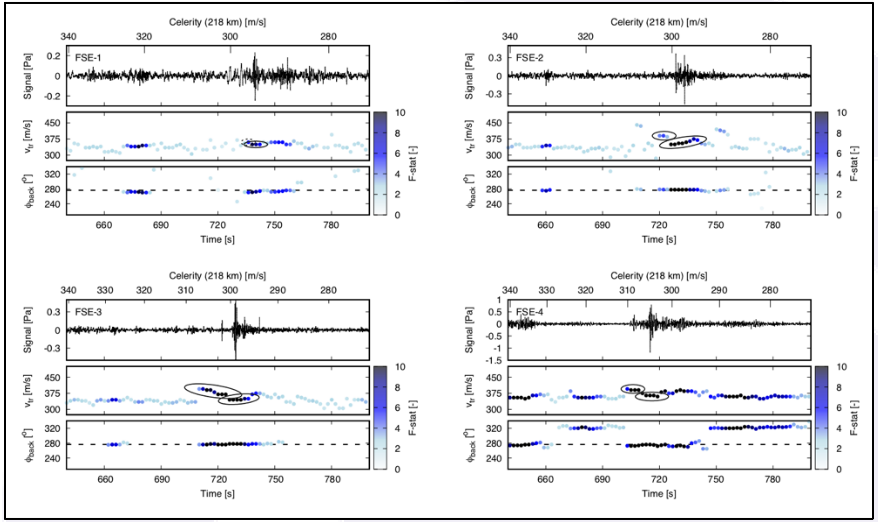
$$P(A_m|\{D_1, D_2, \ldots D_K\}, S_0) = \frac{\prod_k P(D_k|S_0, A_m)}{P(\{D_1, D_2, \ldots, D_K\})} P_0(A_m)$$

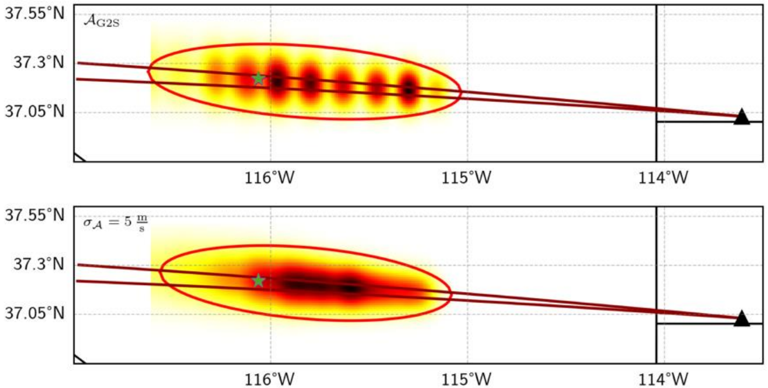


Atmospheric perturbation ensembles computed using *stochprop* software (see Blom et al., 2023)

Example – Single-Station Multi-Phase Detections

- Tropospheric and stratospheric phases were identified at St George during the FSE explosions
- A localization estimate has been computed for FSE-2 using tropospheric and slow stratospheric phases
- 95% confidence region of 5400 km^2





	Ground Truth	Back Projection
Location	37.221°, -116.061°	37.224°, -116.030°
Origin time	20:06:00	20:06:06
σ_{NS}, σ_{EW}, σ_{τ}	-	13.8 km, 20.1 km, 63 sec

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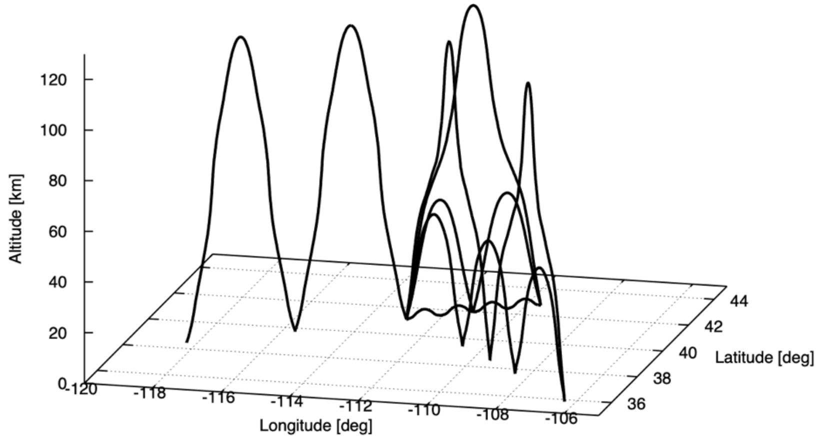
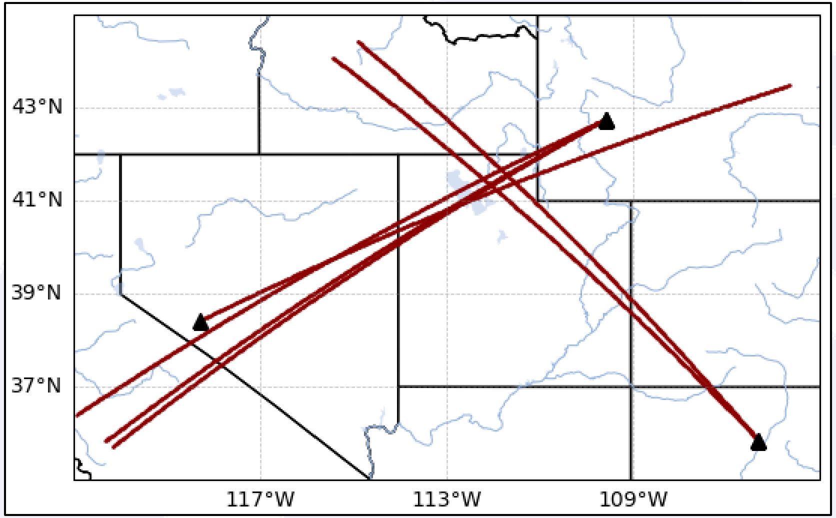
Example – Multi-Station Regional Detections

- Synthetic event was constructed using a reference set of stations in the western US
  - Source at Utah Test and Training Range (UTTR)
  - Stations DLIAR (LANL), PDIAR (Pinedale, WY), and NVIAR (Mina, NV)





- Propagation times and direction-of-arrival information computed using eigenray analysis
- Detection JSON file constructed usable in InfraPy BISL and in-development back-projection localization methods

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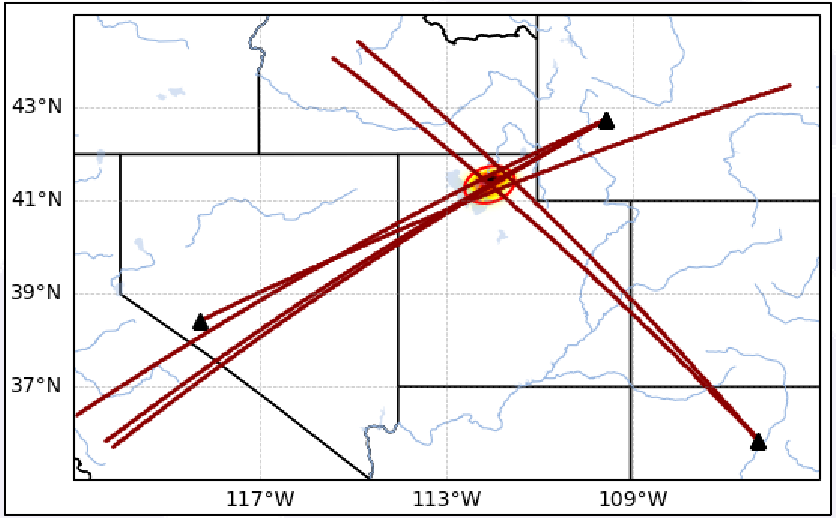

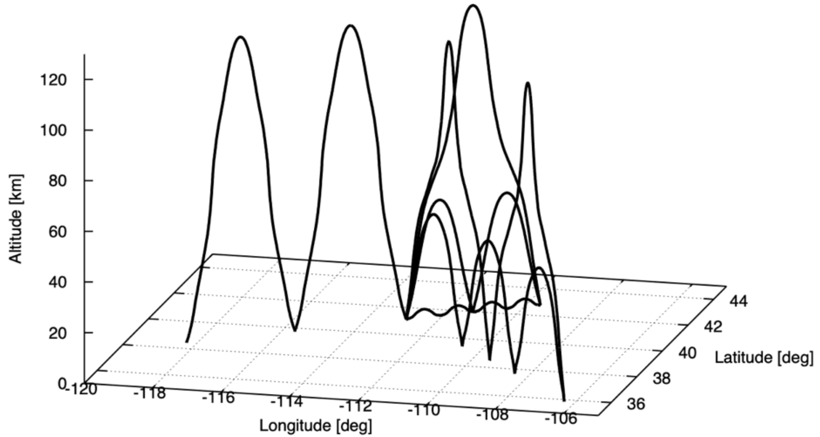

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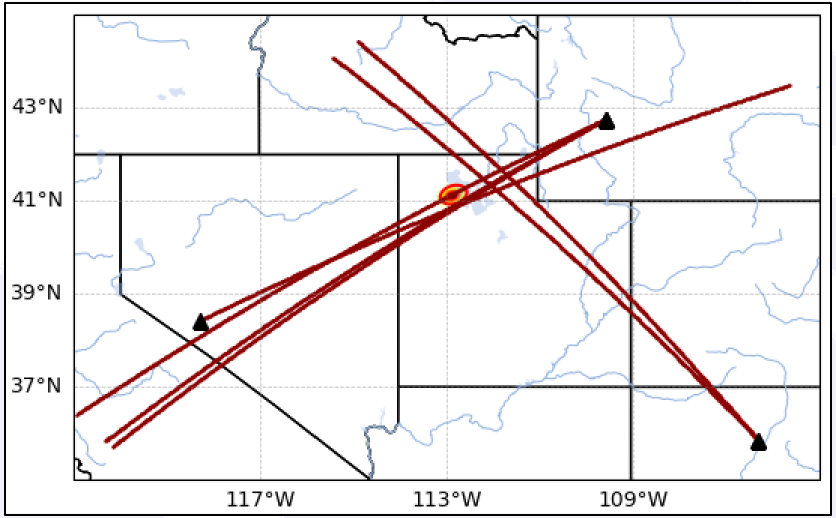

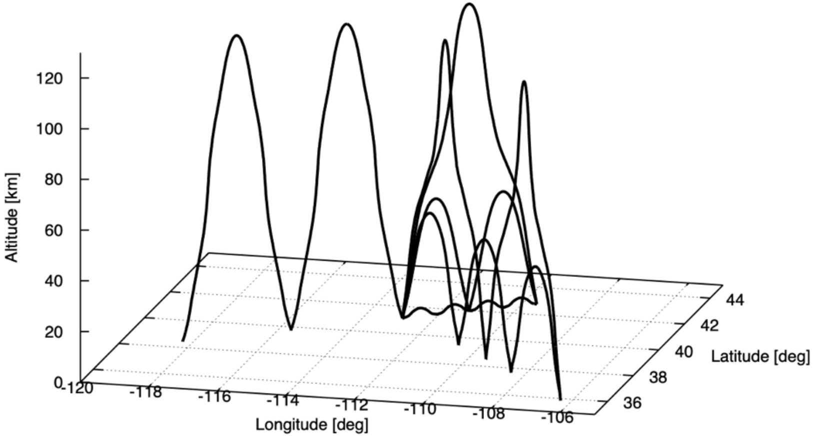

- Propagation times and direction-of-arrival information computed using eigenray analysis
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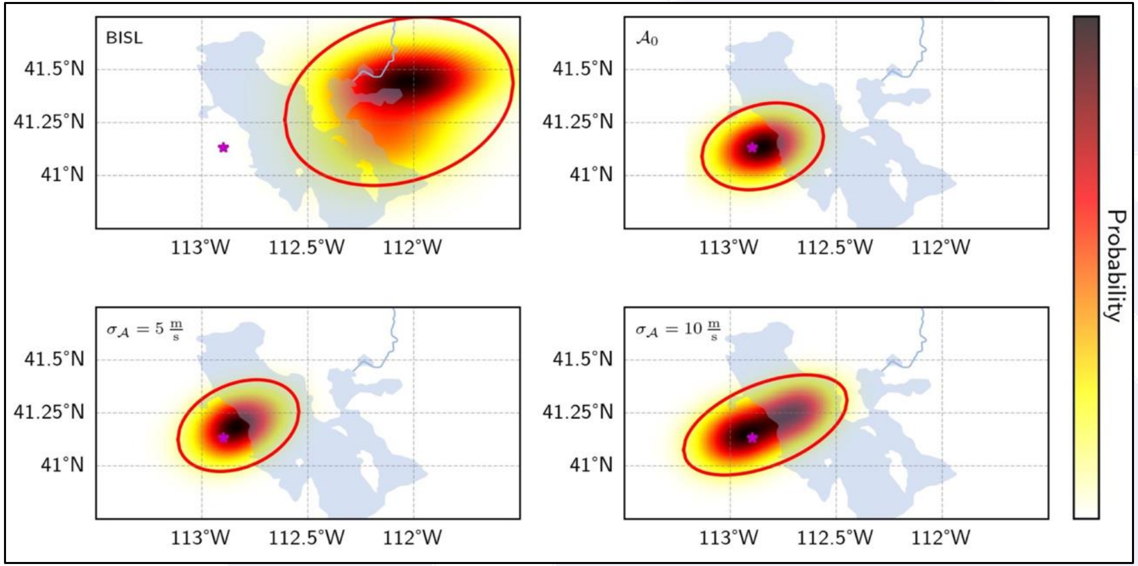
Example – Multi-Station Regional Detections

- BISL lacks crosswind corrections, so the estimated source is displaced in the direction of the stratospheric winds (cross wind impact is underestimated)
- Back projection methods produce a more accurate estimate and reduce area of the 95% confidence ellipse (2200 km^2 vs. 8100 km^2)



	Ground Truth	BISL	A_0	5 m/s uncertainty	10 m/s uncertainty
Location	41.131°, -112.896°	41.351°, -111.98°	41.138°, -112.847°	41.190°, -112.827°	41.194°, -112.835°
Origin time	12:06:00	12:08:02	12:06:14	12:06:24	12:06:23
σ_{NS}, σ_{EW}, σ_τ	-	20.6 km, 20.9 km, 83 sec	10.6 km, 11.2 km, 38 sec	11.2 km, 11.2 km, 36 sec	15.0 km, 12.2 km, 43 sec

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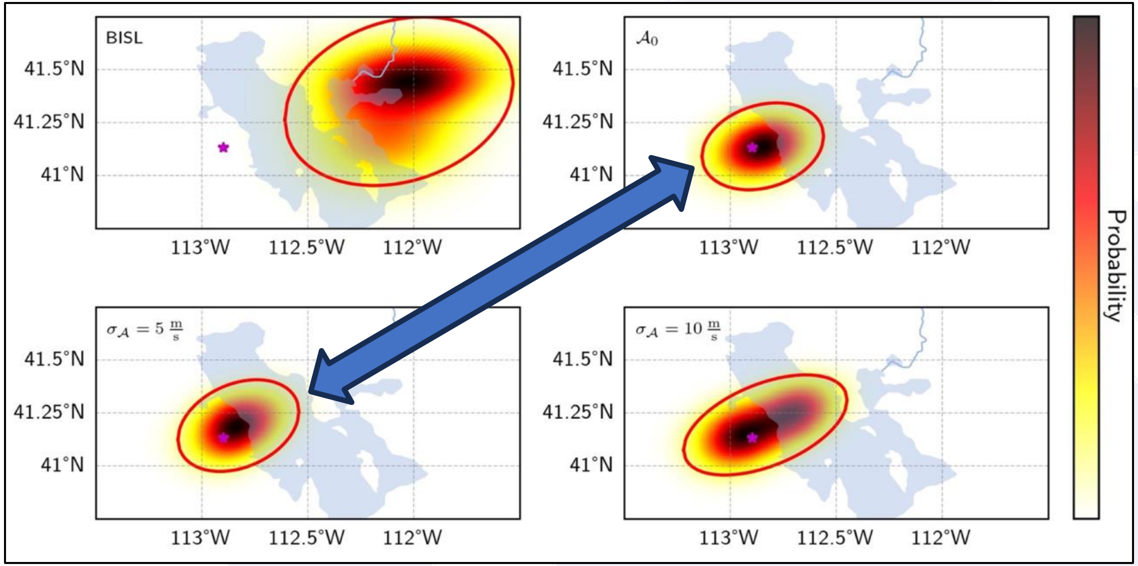
Localization Confidence Trends

- There are minimal differences between the A_0 estimate and that obtained for $\sigma_v = 5$ m/s
- This implies that uncertainty introduced by finite detection SNR (f-stat = 25) dominates compared with that introduced by this atmospheric uncertainty



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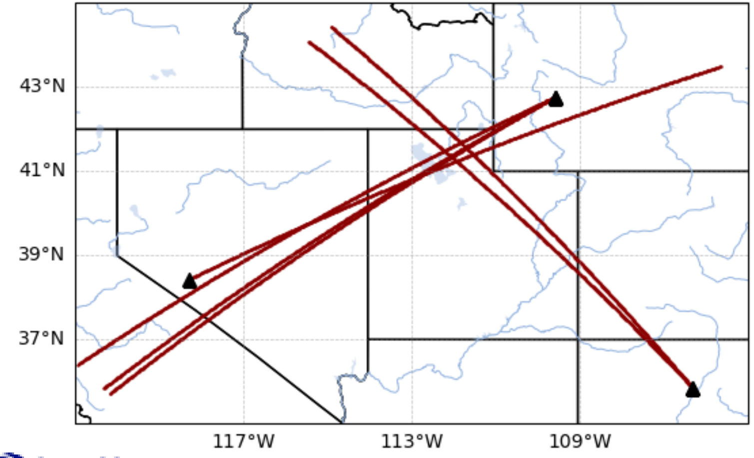
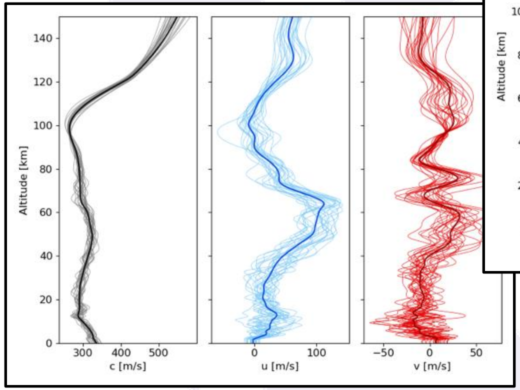
Origin time	12:06:00	12:08:02	12:06:14	12:06:24	12:06:23
$\sigma_{NS}, \sigma_{EW}, \sigma_{\tau}$	-	20.6 km, 20.9 km, 83 sec	**10.6 km, 11.2 km, 38 sec**	**11.2 km, 11.2 km, 36 sec**	15.0 km, 12.2 km, 43 sec

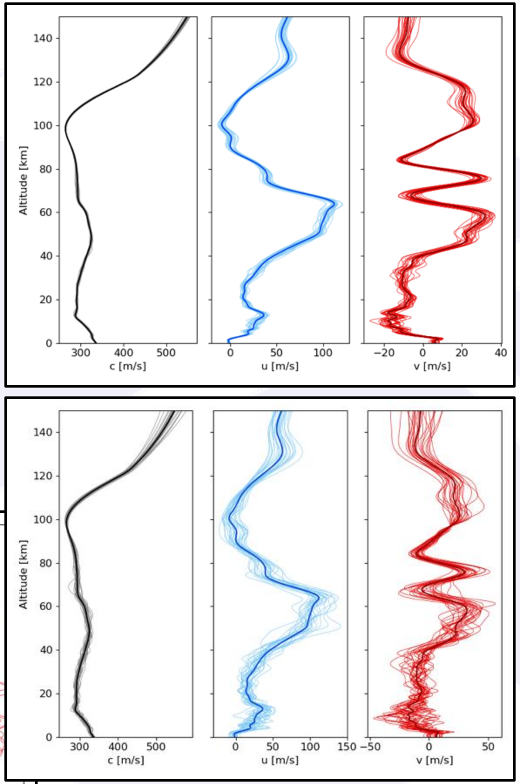
Localization Confidence Trends

- Modified detection sets and atmosphere ensembles have been constructed to investigate the impact of atmospheric uncertainty compared with detection quality
  - Detection sets with f-stat values ranging from 5 to 200
  - Atmosphere with σ_v values between 4 m/s and 20 m/s

- Recently introduced *stochprop* Python tools enable construction of such ensembles







σ_v = 4, 12, and 20 m/s ensembles

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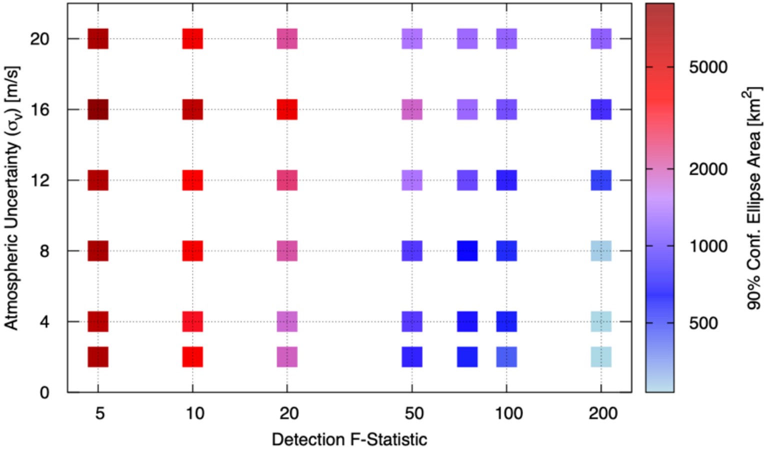
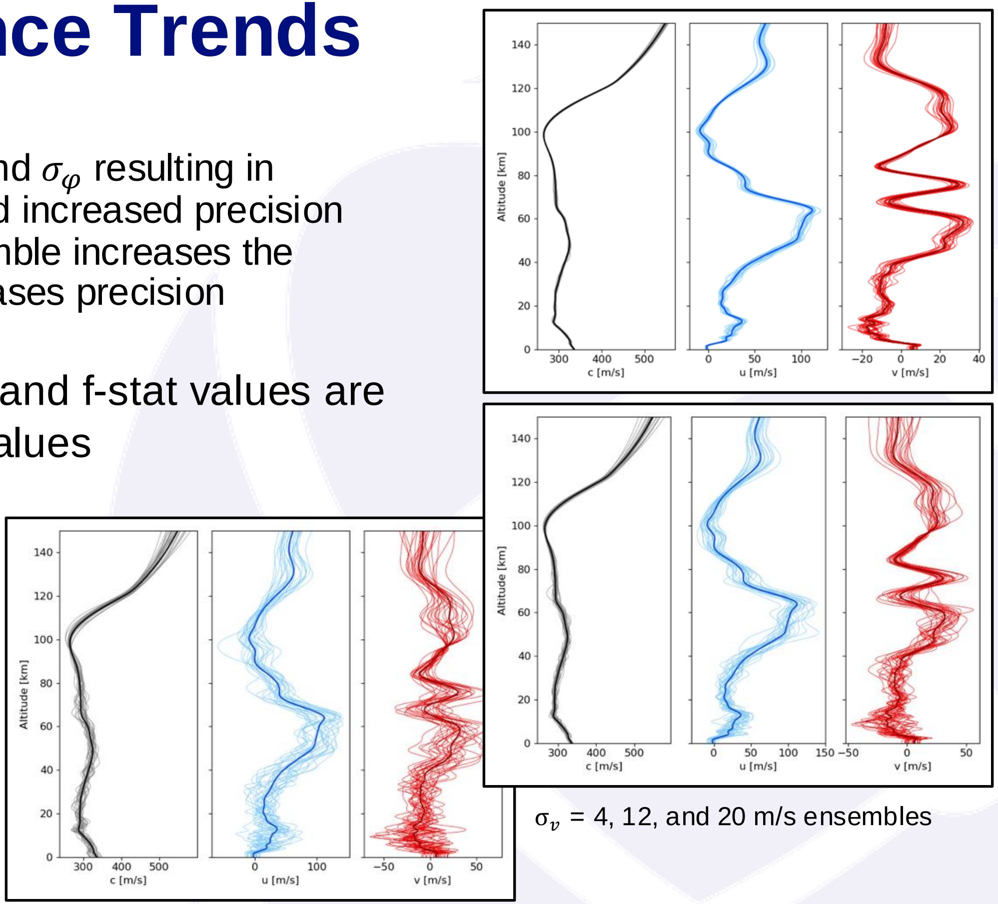
Localization Confidence Trends

- General trends are as expected:
  - Larger f-stat values decrease σ_ϑ and σ_φ resulting in smaller confidence ellipse area and increased precision
  - Larger σ_v in the atmosphere ensemble increases the uncertainty ellipse area and decreases precision

- Limited variations when σ_v is small and f-stat values are large, but σ_v dominates for larger values



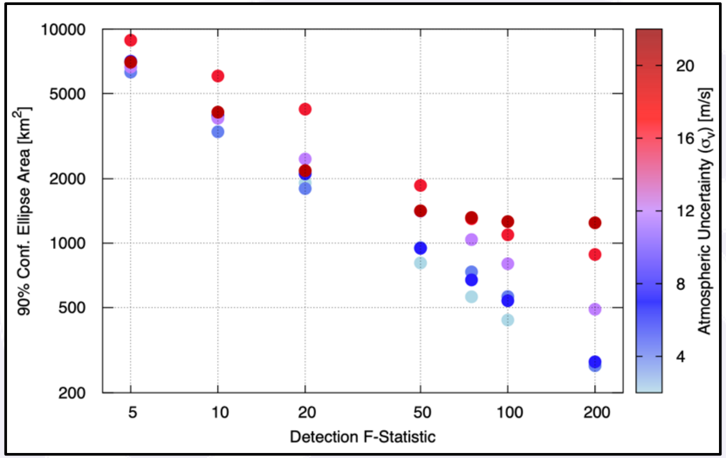


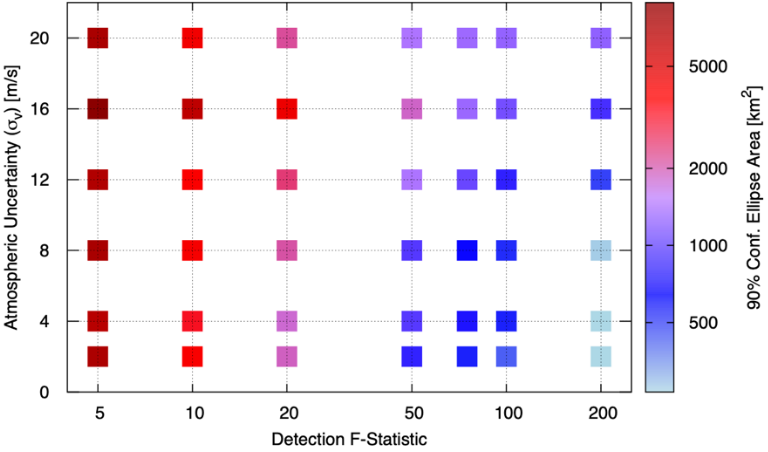
σ_v = 4, 12, and 20 m/s ensembles

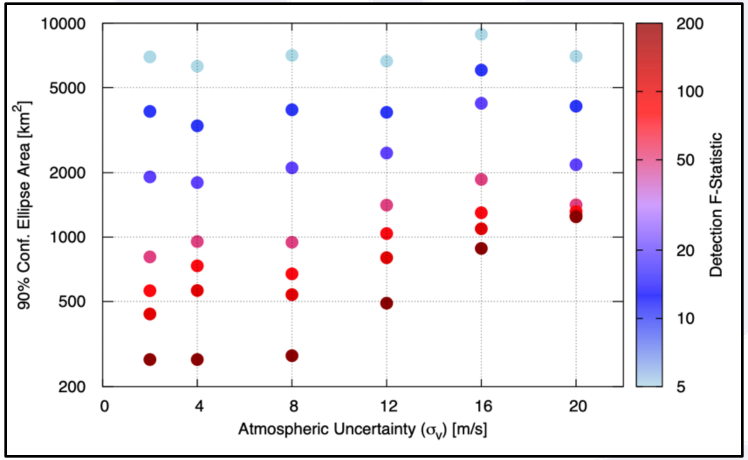
Localization Confidence Trends

- Relative variations are small for small f-stat values, but increase and become more strongly dependent on σ_v for f-stat > 20

- Confidence is nearly constant for small σ_v and increase starting at values around σ_v values of 8 – 12 m/s







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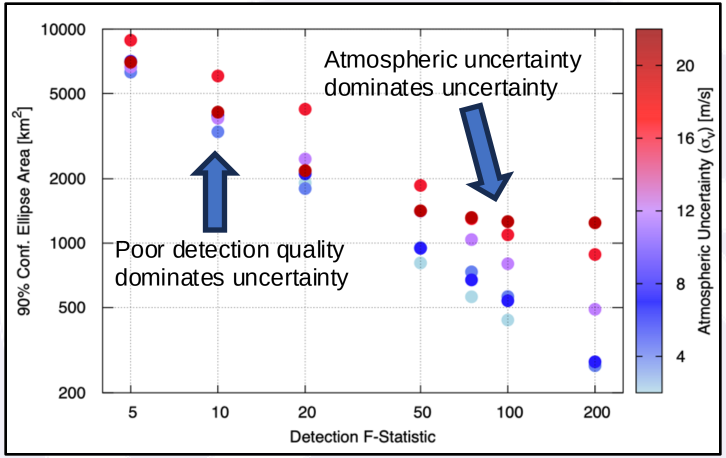
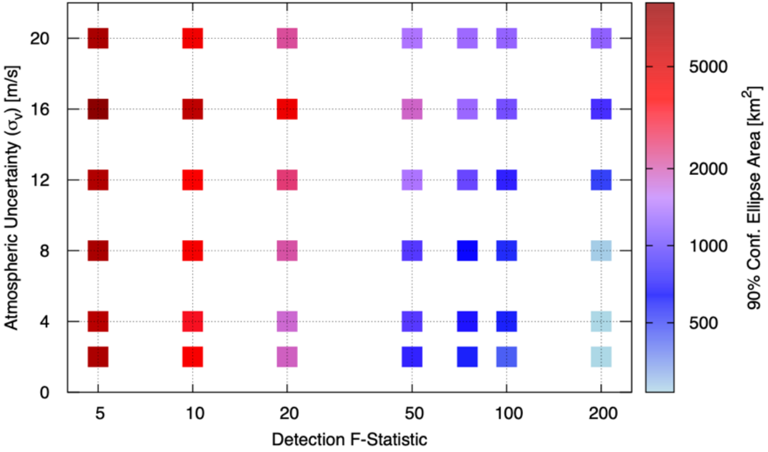
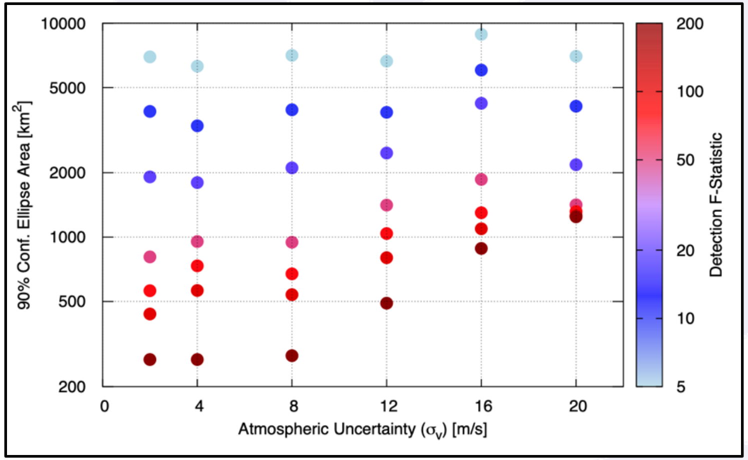
- Confidence is nearly constant for small σ_ν and increase starting at values around σ_ν values of 8 – 12 m/s







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Summary and Ongoing R&D

- **Auxiliary parameters** investigated for solving the Jacobian and computing geometric spreading can also be utilized in eigenray identification, weakly non-linear waveform evolution, and in localization likelihood construction.
- The detection SNR can be used to quantify **uncertainty in back projection launch angles**
- Application of the localization method to a synthetic **multi-station event** as well as an actual **single-station multi-phase** analysis show promising results
- Sensitivity analysis related to the detection quality (f-stat) and atmospheric uncertainty (via ensemble analysis) shows complicated behavior in the **uncertainty budget** → continued sensitivity analysis is needed

Shameless Promotion

LANL SMEs authored an article in a recent issue of Acoustics Today detailing the history and current landscape of infrasound R&D for Nuclear Non-Proliferation.

https://acousticstoday.org/listening-for-a-boom-you-cant-hear/

FEATURED ARTICLE

Listening for a Boom You Can't Hear

Philip Blom and Jordan Bishop

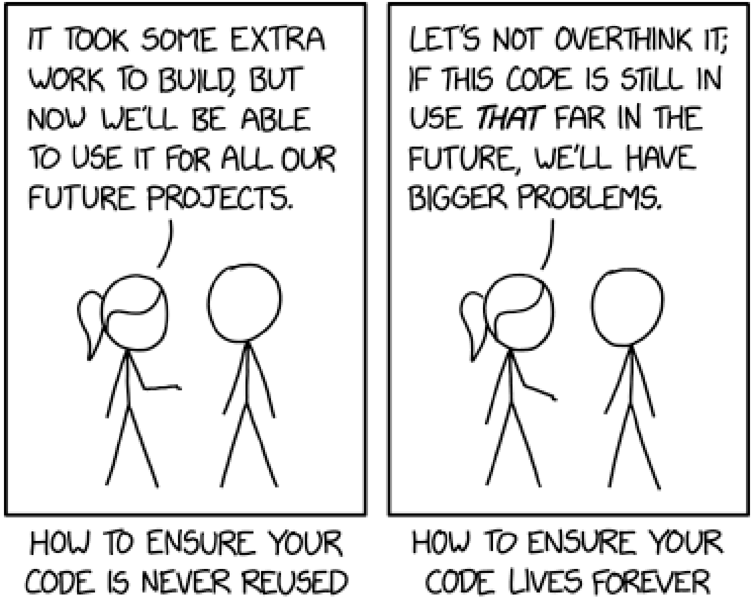
Introduction

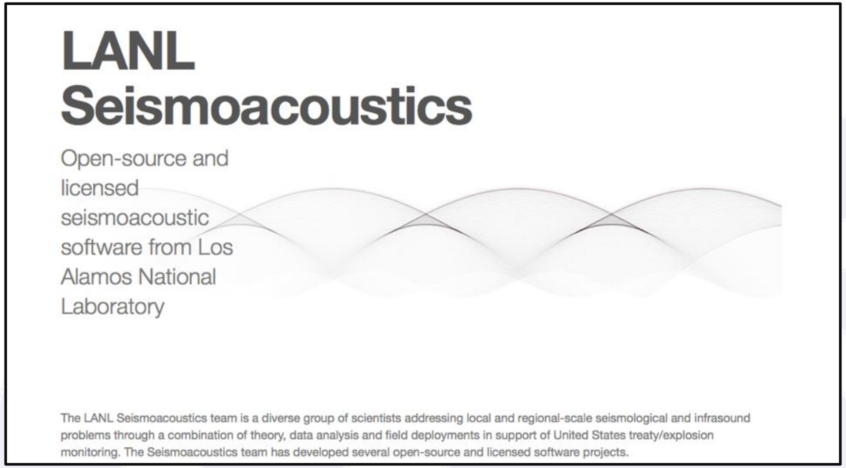
On the morning of July 16, 1945, seismic and acoustic signals were recorded across the southwestern United States from an excessively energetic event. These signals emanated from a location roughly 50 km south of Socorro, New Mexico. Microbarographs 45 km away near San Antonio, Texas, recorded a local overpressure from the event of more than 780 Pa (Manley et al., 1945). This overpressure exceeds 150 dB sound pressure level and is equivalent to being in the immediate vicinity of a shotgun blast or large firework explosion. Seismic and acoustic signals from this event were observed more than 1,000 km away near Mount Wilson in southern California and numerous other locations across the southwestern United States (Gutenberg, 1946).

The source of these observations was the test of an implosion-design plutonium bomb called Trinity (see bit.ly/Blom1) that was conducted as part of the Manhattan Project to develop a nuclear bomb. The bomb design was the product of years of research by J. Robert Oppenheimer and other scientists at Los Alamos Laboratory, Los Alamos, New Mexico. The device was nicknamed "the gadget" and released an explosive energy equivalent to 21,000 tons of TNT (US Department of Energy [DOE], 2015). The fireball pro-

Figure 1. *The fireball produced by the Trinity nuclear test. Photo by Jack W. Aeby, captured as part of the Manhattan Project. Available at bit.ly/3ySMzMD.*

Questions?





https://github.com/LANL-Seismoacoustics

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