Towards Data-Adaptive Beamforming: Generalized Least Squares and other Subspace Methods

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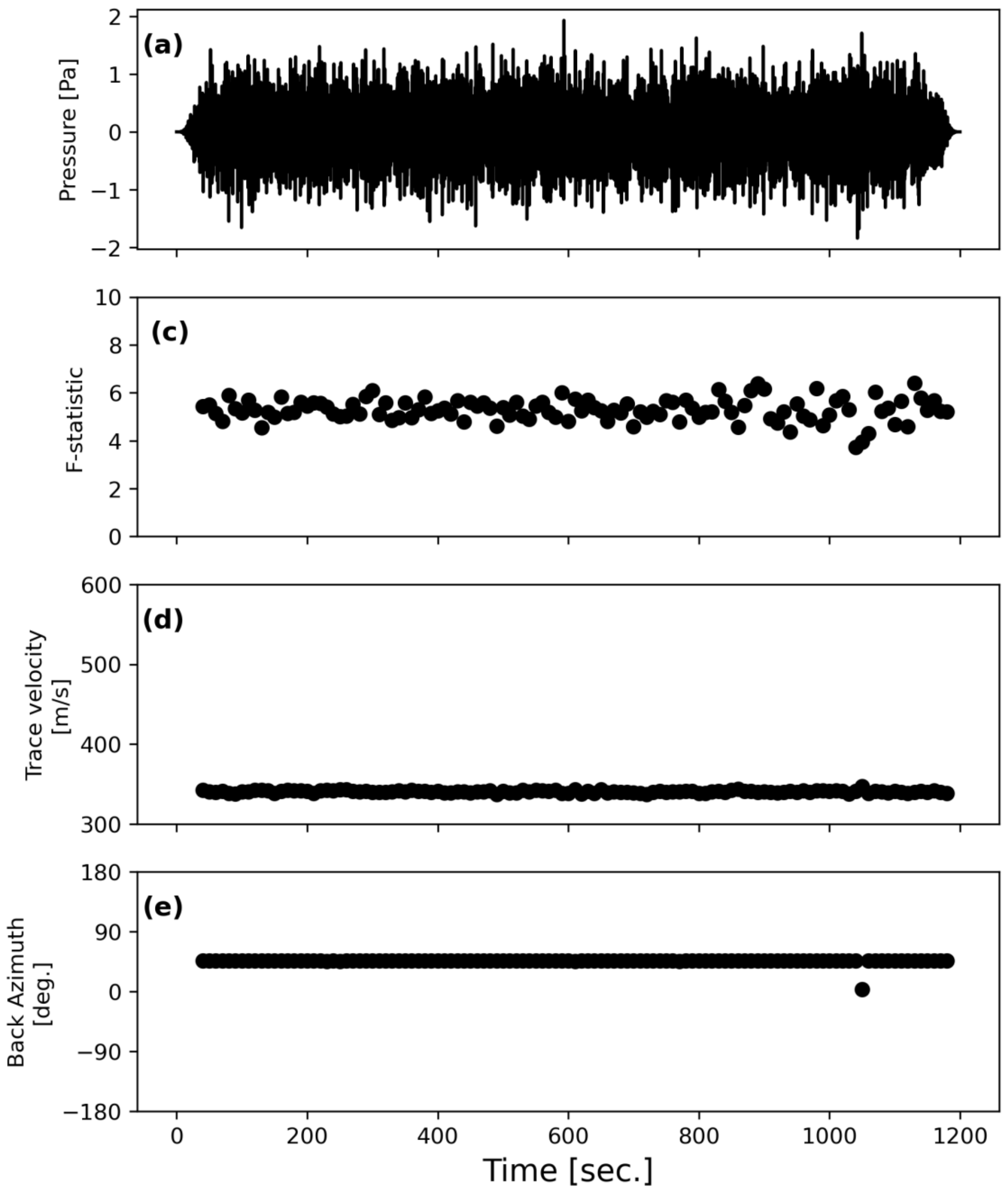
Coherent Backgrounds

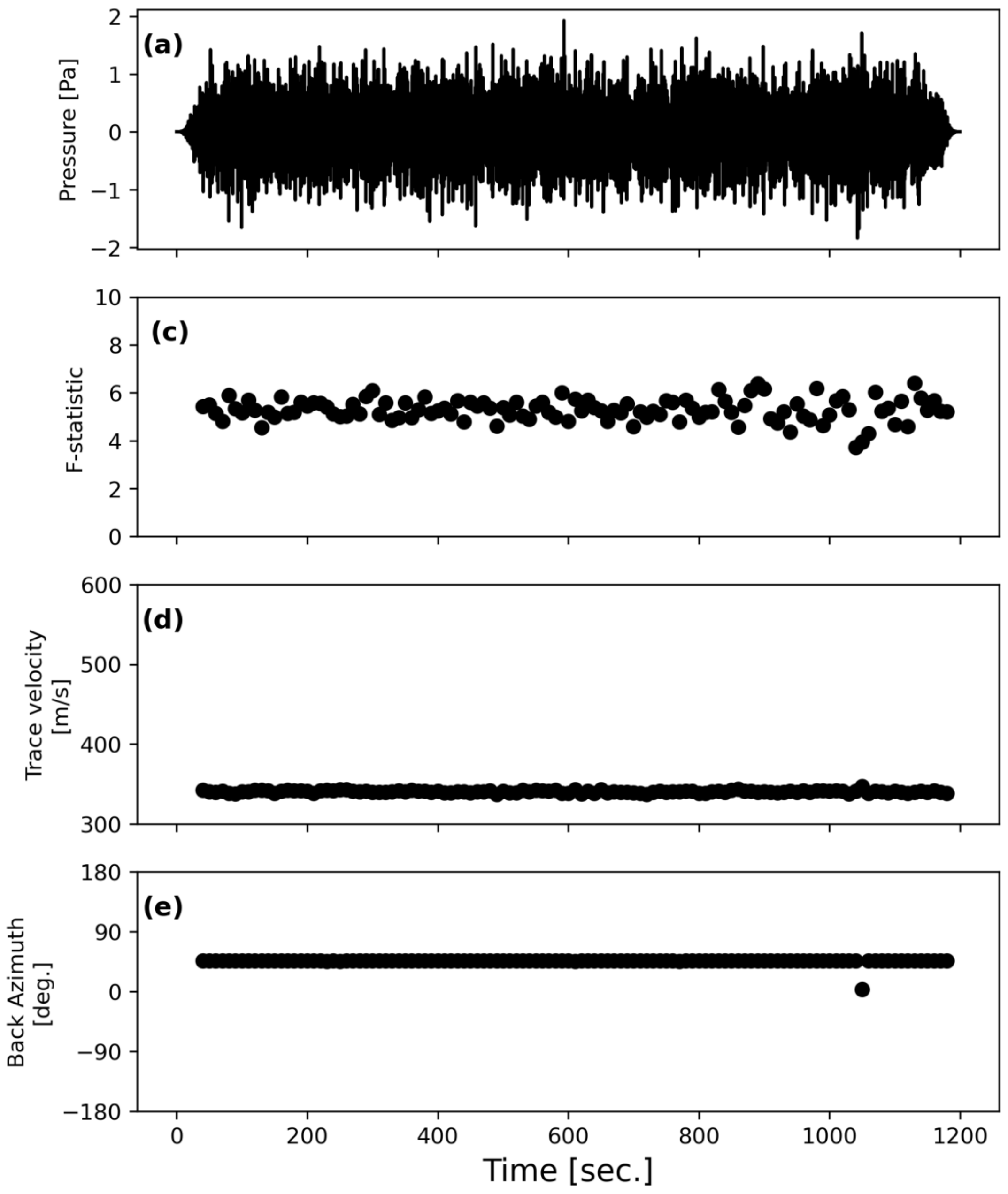
- When processing real infrasound data, it is not uncommon for multiple signals to appear simultaneously, especially when one signal repeats or has an extended duration, for example, microbaroms, wind farms, etc.

- We can consider these coherent backgrounds as **natural jamming signals**.



Coherent Backgrounds

- For the detection problem, the adaptive F-detector (Arrowsmith et al., 2009) uses a moving window to raise or lower detection thresholds across time.
- For parameter estimation (trace velocity and back azimuth), tools like fk-CLEAN and MUSIC can be used to separate multiple coincident infrasound signals.
- The communications literature has a well developed theory for overcoming jamming signals (Van Trees, 2002). Here we follow this approach and leverage the **generalized least squares method**.



Generalized Least Squares (GLS)

- A model for a plane wave signal crossing an array of spatially distributed microphones is

$$\vec{X}(f) = F(f)\vec{\Phi}(f) + \vec{\epsilon}(f), \tag{1}$$

  where $\vec{X}$ is recorded data, F is the signal amplitude, $\vec{\Phi}$ is a steering vector, and $\vec{\epsilon}$ is random noise.
- A goal of array processing is to estimate the signal of interest F using a weighted sum of the recorded data $\vec{X}$.

$$F \simeq= \vec{w}^{\dagger}\vec{X}. \tag{2}$$

- Maximizing the beam power with a normalized weight vector ($|\vec{w}| = 1$) leads to the Bartlett beam,

$$P_B = \vec{w}^{\dagger}\mathbf{S}\vec{w}, \tag{3}$$

  where $\mathbf{S} = \vec{X}\vec{X}^{\dagger}$ is the data covariance matrix.

Generalized Least Squares (GLS)

- We can generalize the signal model to include K coherent plane wave signals as

$$\vec{X} = F\vec{\Phi} + \vec{N}. \tag{4}$$

$$\vec{N} = \sum_{k}^{K} G_k \vec{\Psi}_k + \vec{\epsilon}.$$

- The noise term $\vec{N}$ has a covariance matrix

$$\mathbf{S_n} = \vec{N}\vec{N}^{\dagger} = \sum_{i}^{K} \sum_{j}^{K} G_i \bar{G}_j \vec{\Psi}_i \vec{\Psi}_j^{\dagger} + \sigma_w^2 \mathbf{I}. \tag{5}$$

Generalized Least Squares (GLS)

- From a maximum likelihood approach, the generalized least squares weight vector $\vec{w}_{GLS}$ is
$$\vec{w}_{GLS} = \left(\vec{\Phi}^\dagger \mathbf{S_n}^{-1} \vec{\Phi}\right)^{-1} \mathbf{S_n}^{-1} \vec{\Phi}. \tag{6}$$
- Thus, the GLS beam power is
$$P_{GLS} = \frac{\left|\vec{\Phi}^\dagger \mathbf{S_n}^{-1} \vec{X}\right|^2}{\left|\vec{\Phi}^\dagger \mathbf{S_n}^{-1} \vec{\Phi}\right|^2}. \tag{7}$$
- With uncorrelated Gaussian noise, $\mathbf{S_n} = \sigma_w^2 \mathbf{I}$, and a normalized weight vector, we recover the Bartlett beam,
$$P_{GLS} = P_B = \vec{w}^\dagger \mathbf{S} \vec{w}. \tag{8}$$

Generalized Least Squares (GLS)

- To better understand how the GLS method works, we apply the matrix inversion lemma to the noise covariance matrix $\mathbf{S_n}$:

$$\mathbf{S_n}^{-1} = \frac{1}{\sigma_w^2}\left[\mathbf{I} - \mathbf{V}\left(\mathbf{G}^{-1} + \mathbf{V}^{\dagger}\mathbf{V}\right)^{-1}\mathbf{V}^{\dagger}\right]; \quad \mathbf{V} = [\vec{\Psi}_1|\vec{\Psi}_2|...|\vec{\Psi}_K]; \tag{9}$$

$$\mathbf{G} = \frac{1}{\sigma_w^2}\begin{bmatrix} |G_1|^2 & \dots & G_1\bar{G}_K \\ \vdots & \dots & \vdots \\ G_K\bar{G}_1 & \dots & |G_K|^2 \end{bmatrix}. \tag{10}$$

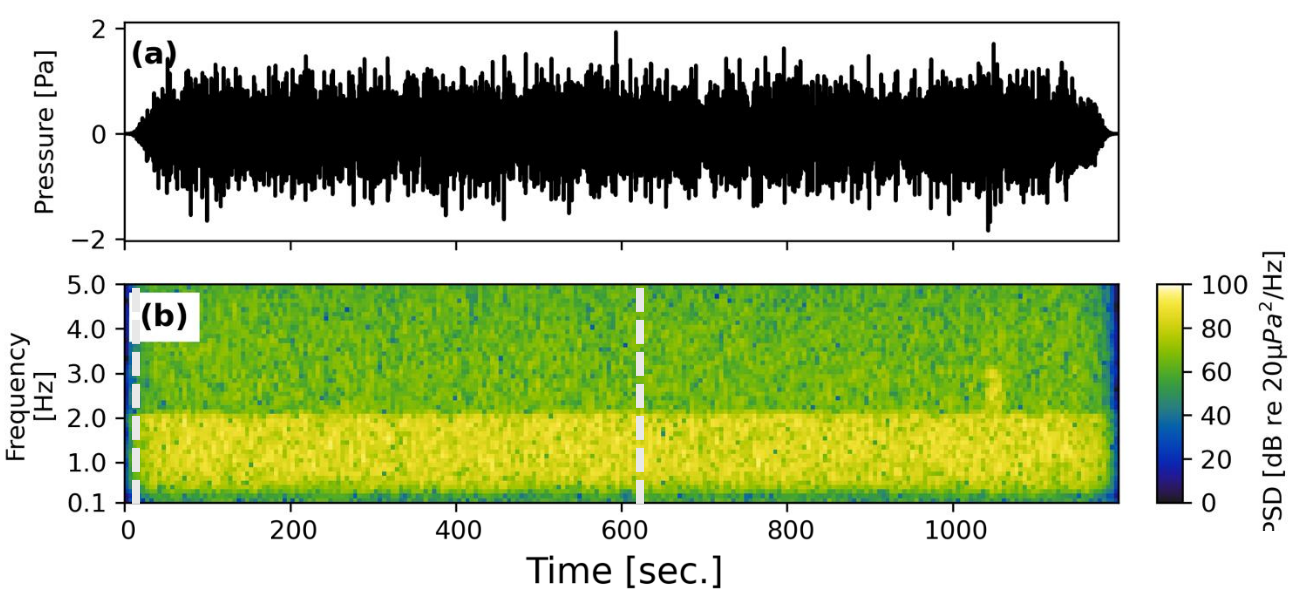
- Thus, with large noise SNR ($|G_k|^2 >> \sigma_w^2$), we obtain an orthogonal projector

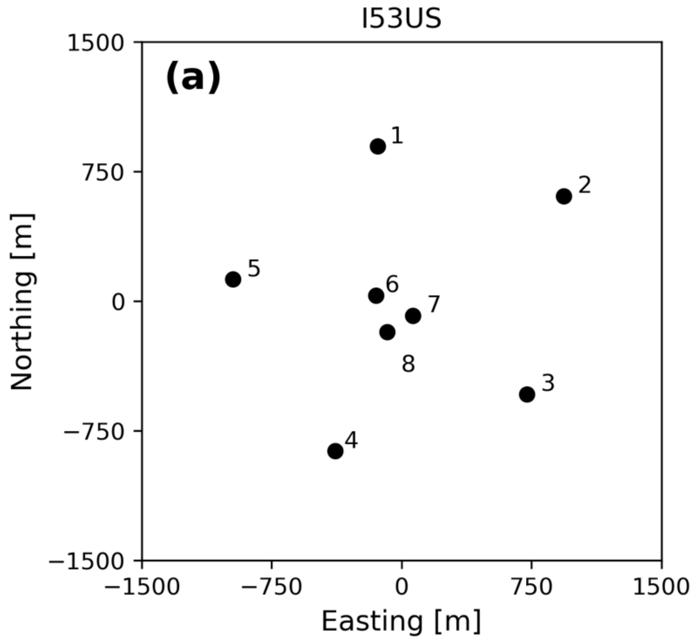
$$\mathbf{S_n}^{-1} = \frac{1}{\sigma_w^2}\left[\mathbf{I} - \mathbf{V}\left(\mathbf{V}^{\dagger}\mathbf{V}\right)^{-1}\mathbf{V}^{\dagger}\right]. \tag{11}$$

Using an estimate of the noise covariance, beamforming occurs in a subspace normal to the span of the noise steering vectors ($\vec{\Psi}_k$), effectively nulling their contribution.

Synthetic Example – Coherent Background Noise

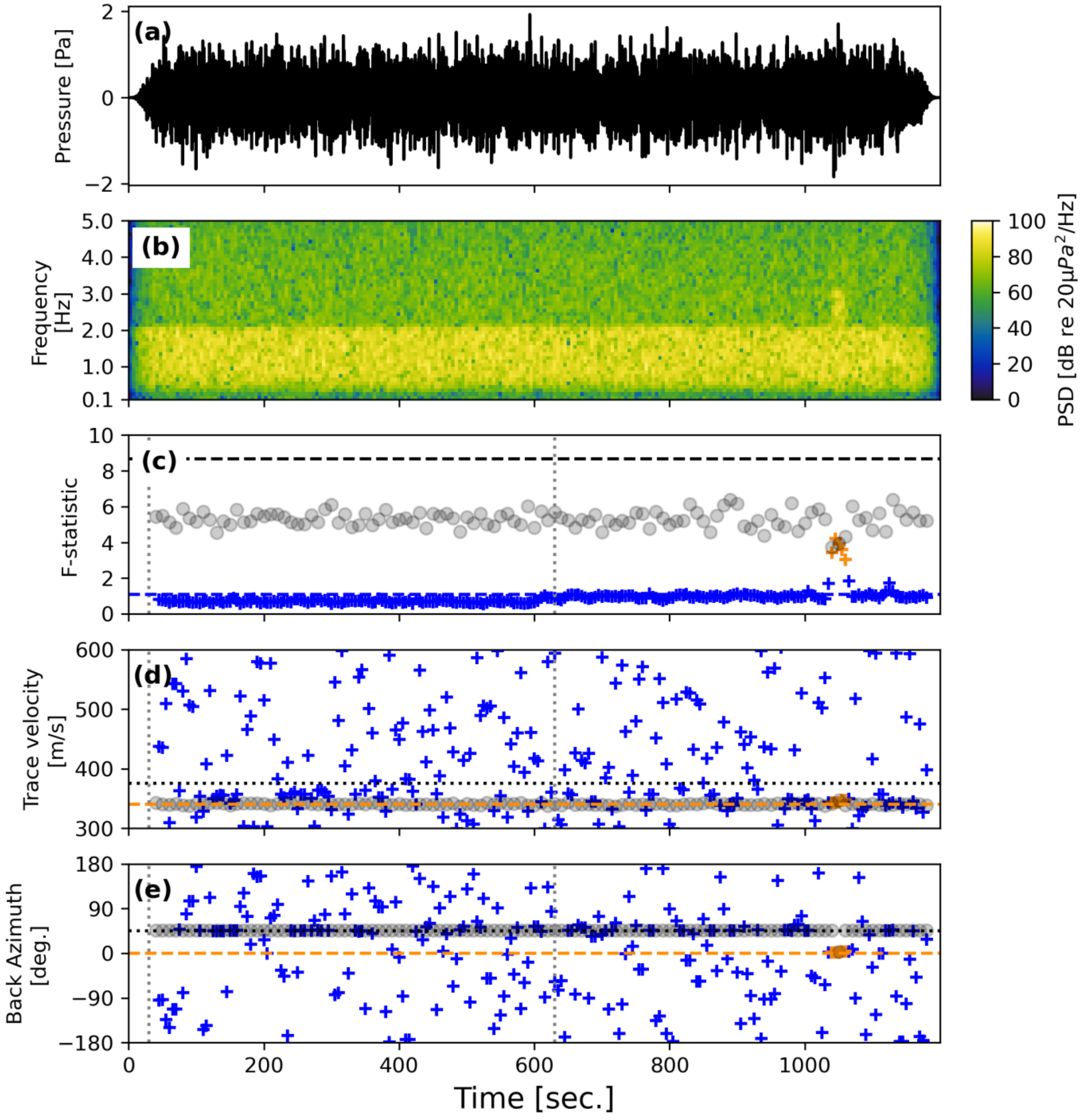
- A low SNR transient signal impinges on an array during a persistent background signal
- Transient signal:
  - back az. = 0°
  - trace vel. = 340 m/s
- Persistent background:
  - back az. = 45°
  - trace vel. = 375 m/s
- A ten minute window was used to characterize the background noise (gray dashed lines).
- Beamforming occurred in 20 second windows with 5 second steps in a 0.5 – 5.0 Hz frequency band.



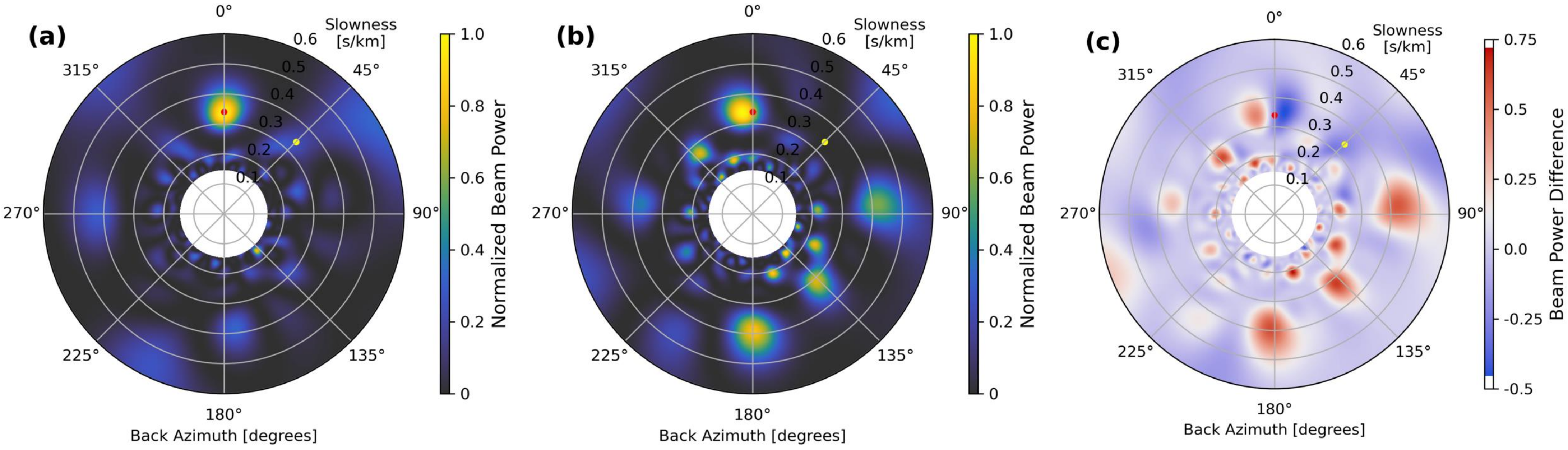


Synthetic Example – Coherent Background Noise

- F-statistic estimates are relatively constant for the Bartlett beam, but a clear peak is visible at the signal arrival time for GLS (orange markers).
- A consistent back azimuth and trace velocity are visible for the Bartlett beam.
- These values are randomized for the GLS processing, which suggests their contributions are nulled.



Interpretation with Array Response Functions

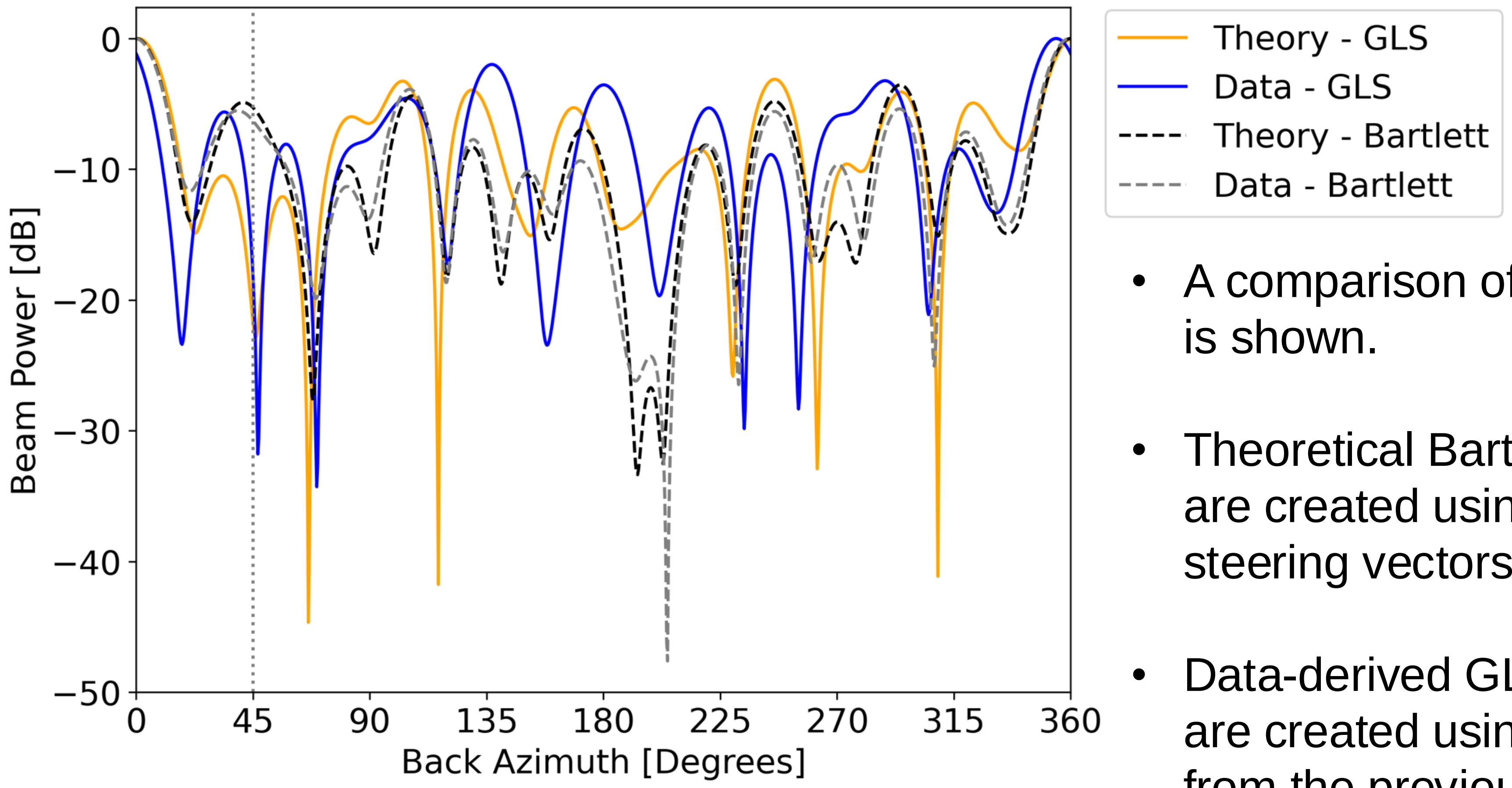


Bartlett GLS Difference

- Array response functions (ARFs) at 0.5 Hz that are steered towards the signal of interest (red dot)

  back azimuth = 0°, trace velocity = 340 m/s.
- The GLS array response function shows a smaller beam response in the direction of the coherent background (yellow dot).
- However, at this frequency, the GLS ARF is biased by about 5°, which drops to 2° at 1.0 Hz. This appears to be an effect of the noise covariance matrix at 0.508 Hz.

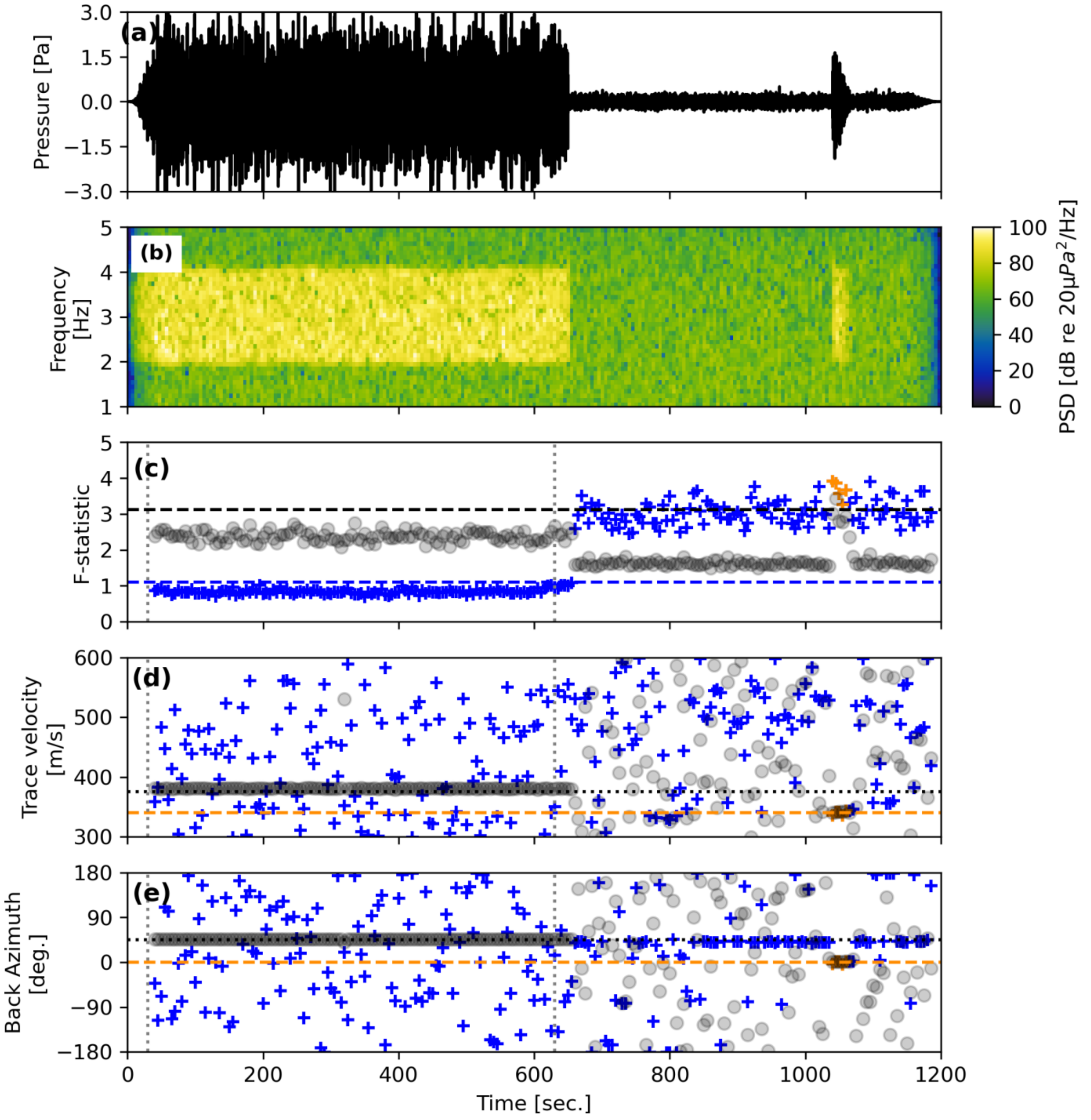
A comparison of beam patterns



- A comparison of the beam patterns at 0.5 Hz is shown.
- Theoretical Bartlett and GLS beam patterns are created using the known signal and noise steering vectors.
- Data-derived GLS and Bartlett beam patterns are created using the noise covariance matrix from the previous example.
- A dip in the response at 45° (the direction of the coherent noise) is shown for both theoretical and data-derived GLS.

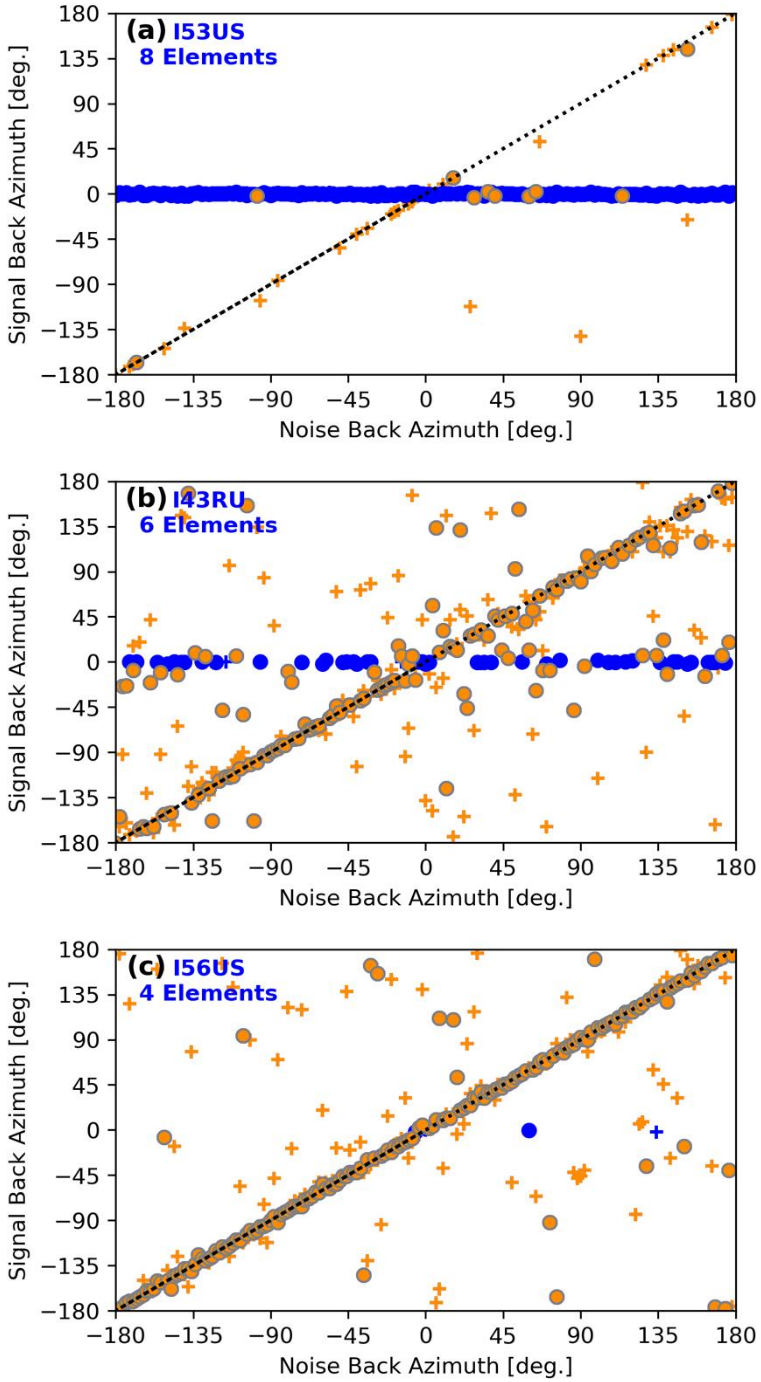
Where GLS breaks down: A mischaracterized background

- The coherent background (back azimuth = 45°, trace velocity = 375 m/s) is no longer present after the background window.
- The GLS F-statistic values increase after the coherent background ends, while the Bartlett beam-derived F-statistic values decrease.
- This effectively hides the signal of interest at 1040 seconds.



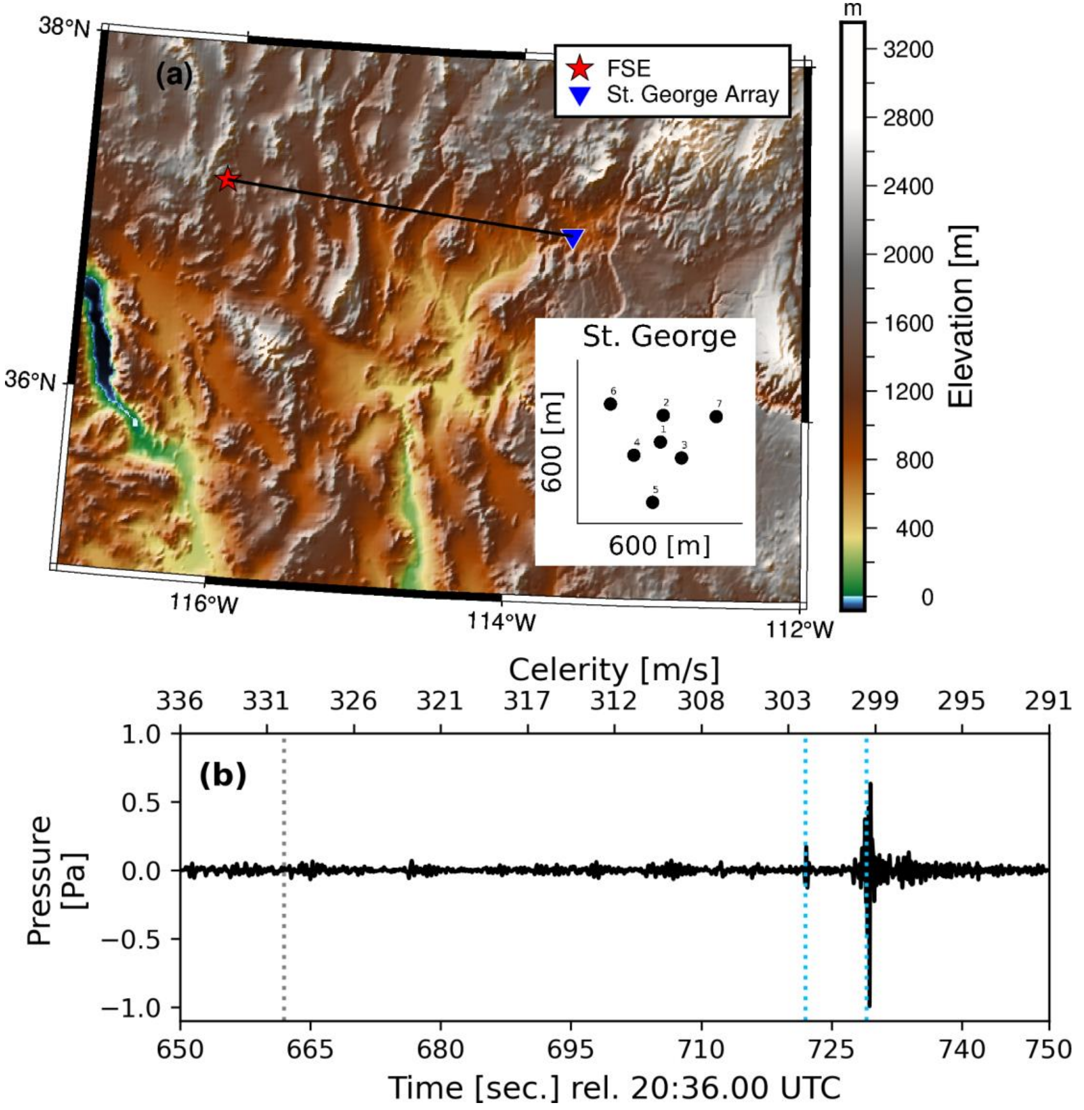
Where GLS breaks down: Small array element numbers

- Using I53US, I43RU, and I56US as representative arrays with 8, 6, and 4 elements.
- The coherent noise was rotated with respect to the signal direction of arrival (0°).
- GLS results within 2° are plotted in blue, with other results plotted in orange. Noise estimates plot along the diagonal line.
- Decreased GLS effectiveness with fewer elements.



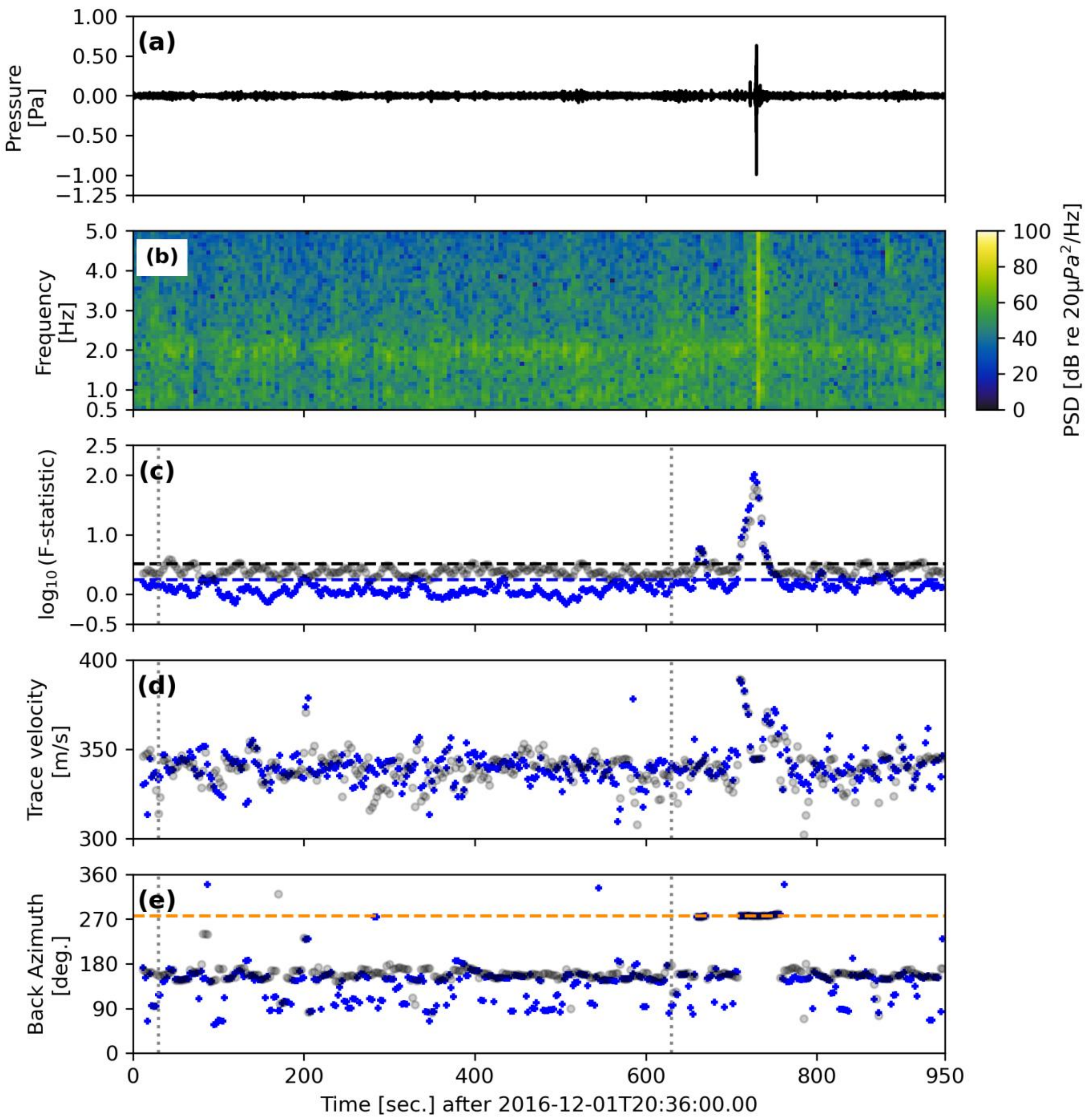
GLS Application to Real Data

- The Forensic Surface Experiment (FSE) was a series of chemical surface explosions at the Nevada National Security Site (NNSS), Nevada, USA in December 2016.
- Infrasound from event three, FSE-3, was recorded on a seven element array located approximately 218 km to the east of the NNSS in St. George, Utah.
- FSE-3 had a 100 kg TNT equivalent yield (chemical), and multiple infrasound arrivals were observed at the St. George array, a low-amplitude tropospheric phase and a stratospheric pair.



GLS Application to Real Data

- Beamforming was performed in a 0.5 – 5.0 Hz band with 20 second windows and a 2.5 second step.
- A 10 minute noise window was chosen.
- The GLS F-statistic values are lower than the Bartlett beam F-statistic values during the coherent noise (before and after the signal), and they are higher during the tropospheric and stratospheric arrivals.
- Note that the GLS trace velocity and back azimuth estimates have higher variance than the Bartlett beam, but they are no longer randomized.



Future Work

- Extension of this method to multiple sources is straightforward, and development of an automated framework is our next goal.
  - At minimum, the noise covariance matrix would need to be updated through time like the adaptive F-detector (Arrowsmith et al. 2009).
- GLS is not mutually exclusive to the fk-CLEAN framework, so perhaps the GLS fk power map could be used with CLEAN postprocessing, or the partially cleaned cross-spectral matrix could be used to construct the GLS weight vector.
- Finally, we note that conventional adaptive beamforming methods that rely on recursive or gradient-based techniques exist in the literature, but, to our knowledge, have yet to be applied to infrasonic processing.

Conclusions

- The generalized least squares (GLS) method outperforms the Bartlett estimator for well characterized noise backgrounds, but the application is limited

- When applied to the Forensic Source Experiment data, the GLS F-statistic estimates had a higher contrast between noise and signal values windows, which may increase detection capability for low signal-to-noise ratio signals.

- More R&D is needed to automate the selection of a high quality noise covariance matrix.

Acknowledgements

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