



Higher – order statistics



Higher-order statistics (spectra) have begun to find wide applicability in many diverse fields, e.g.

- o sonar,
- $\circ\,$ radar,
- o plasma physics,
- o biomedicine,
- o seismic data processing,
- o image reconstruction,
- o harmonic,
- o harmonic retrieval,
- o time-delay-estimation,
- o adaptive filtering,
- o array processing, and
- o blind equalization.

The second, third and fourth-order cumulants of zero-mean x(t), which follow from (2) and (3), are $C_{2,x}(\tau) = E\{x(t)x(t+\tau)\}$ $C_{3,x}(\tau_1, \tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\}$ $C_{4,x}(\tau_1, \tau_2, \tau_3)$ $= E\{x(t), x(t+\tau_1), x(t+\tau_2)x(t+\tau_3)\}$ $- C_{2,x}(\tau_1)C_{2,x}(\tau_2 - \tau_3) - C_{2,x}(\tau_2)C_{2,x}(\tau_3 - \tau_1)$ $- C_{2,x}(\tau_3)C_{2,x}N$

If a random process is symmetrically distributed, then its third-order cumulant equals zero; hence, for such a process we <u>must use fourth-order cumulants</u>.

- o Laplace,
- \circ Uniform,
- o Gaussian, and
- o Bernoulli Gaussian
- distributions are symmetric [1].

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Flow chart of the statistical filter and ToA estimation





Figure 1. Flow chart of statistical 4th order cumulant filter and algorithm for on-site ToA estimation [2,3].

Figure 2. Two gun fire noisy and distorted events. a – filtered data, b – source data, and c – filtered data, and two ToA (Time of Arrival) estimates.



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Statistical and wavelent filtration of the noisy and distorted signals





Figure 3. Signal filtering. a – Filtered signal using a statistical 4th order cumulant filter (or just statistical filter), b –Source data from seismic sensor [4], and c -wavelet decomposition of the signal (Daubechies wavelet coefficient of 7th order [5] (D₇).

Figure 4. By applying a statistical filter, the noise and distortion of the acoustical signal is significantly removed, see figure (a), and figure (b) show source data.



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Noise and distortion removal, and ToA estimation on-site





Figure 5a. Case of very distorted and noisy acoustic signal which originate from gun fire.

Figure 5b. The signal from Fig. 5a after filtering with statistical filter. In addition, algorithm for on-site estimation ToA is applied and ToA value is successfully estimated [3].

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INTRODUCTION

OBJECTIVES

METHODS/DATA

RESULTS

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Conclusions



- A method of filtering very distorted and noisy signals, based on 4th order cumulant, is developed, i.e. a so-called statistical filter.
- The algorithm for estimation the time of arrival (ToA) of the wave front is successfully applied to the signals filtered in this way.
- The statistical filter significantly improves the SNR (Signal to Noise Ratio) of very distorted and noisy signals.
- $\circ~$ The measurement noise is removed from the signal.
- Possible applications in acoustics, hydroacoustics, infrasound, seismic, radar, and other areas.
- Filtrating and ToA estimate is possible to do on-site, and it is thus possible to significantly reduce the load on the data transmission network by sending only estimated ToA values, and/or parts of data with useful content.



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- Jerry M. Mendel, Tutorial on Higher Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical Results and Some Applications, PROCEDINGS OF THE IEEE, VOL. 79, NO. 3, p 278-305, March 1991.
- Miodrag S. Vračar and Ivan Pokrajac, "Application of the algorithm for time of arrival estimation of N-waves produced by projectiles of different calibers", *Acoustics '17 Boston*, 173rd Meeting and 8th Forum Acusticum, Signal Processing in Acoustics: Paper 5pEA9, Boston, Massachusetts, 25-29 June 2017.
- Ivan P. Pokrajac, Miodrag Vračar, Tamara Šević, Vasilija Joksimović, "Position Determination of Acoustic Source Using Higher-Order Spectral Analysis for Time of Arrival Estimation", *Scientific Technical Review*, Vol. 69, No. 3, pp 41-48, 2019.
- 4. <u>https://www.zapsplat/sound-effect-category/war-and-weapons</u>, 19. 05. 2021.

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5. Daubechies I., Ten Lectures on Wavelets, SIAM, Capital City Press, (1992).