

# SnT 2023

CTBT: SCIENCE AND TECHNOLOGY CONFERENCE

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## Testing and Design of Discriminants for Local Seismic Events Recorded During the Redmond Salt Mine Monitoring Experiment in Utah

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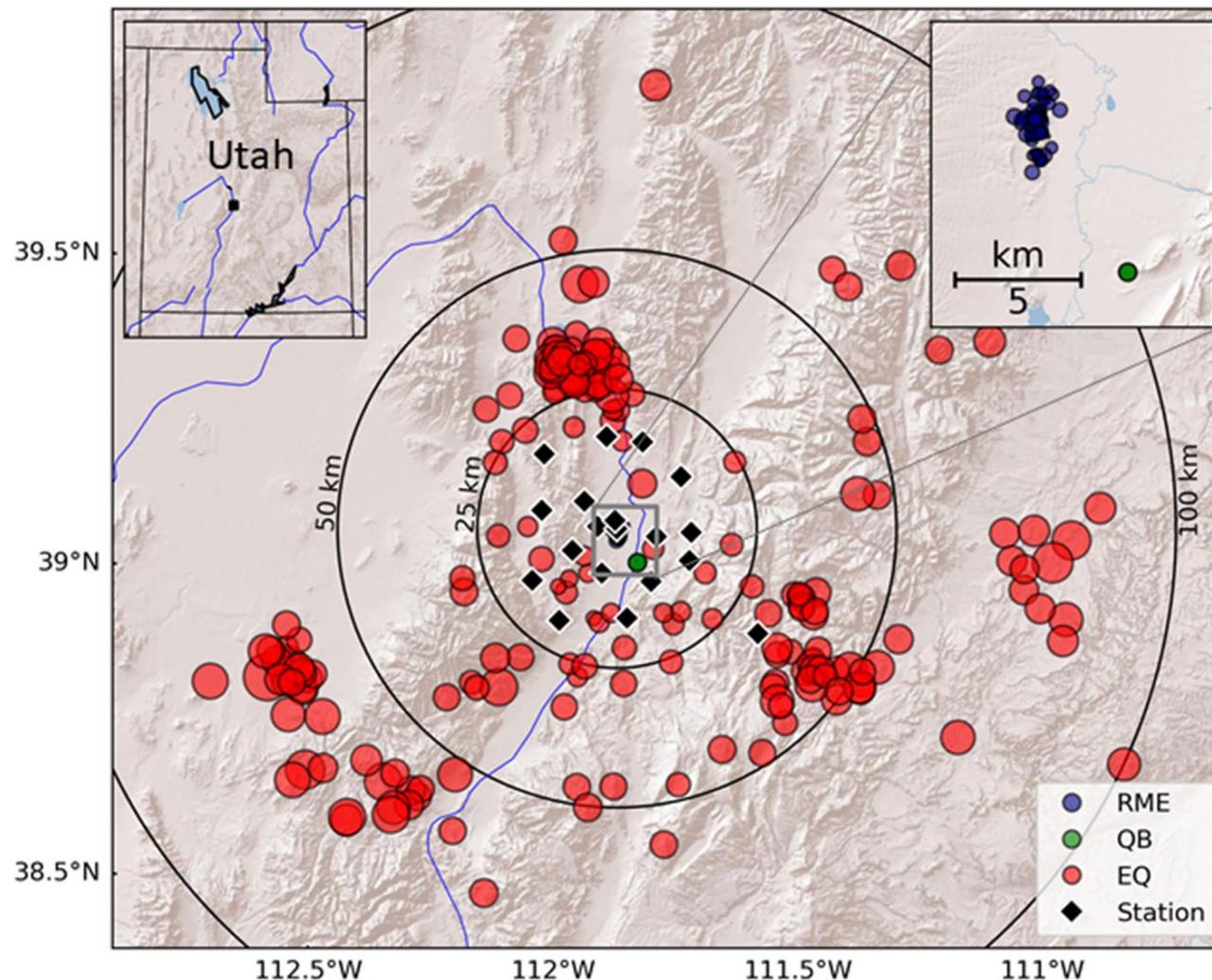


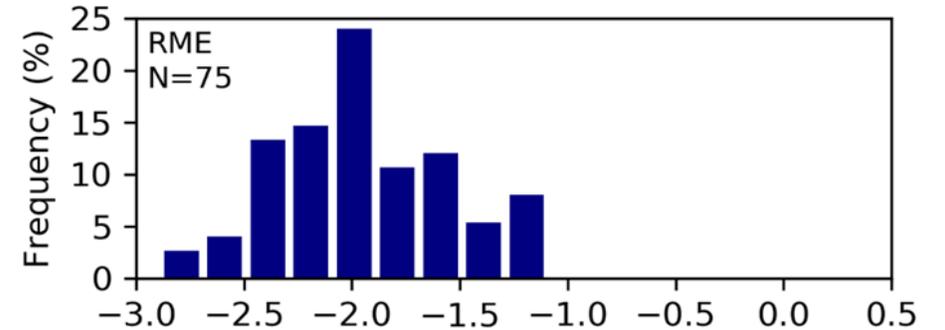
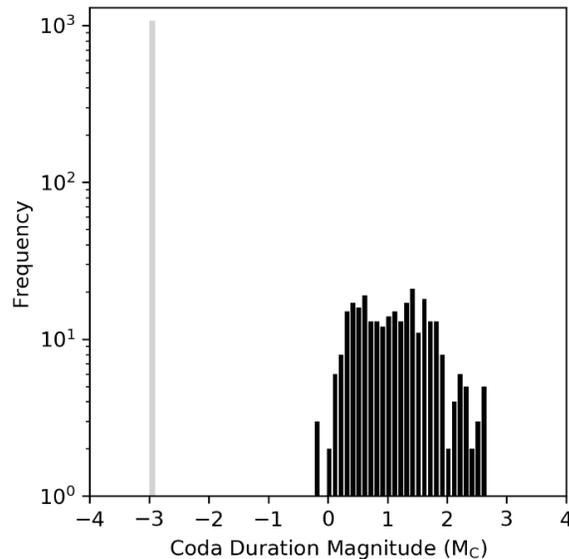
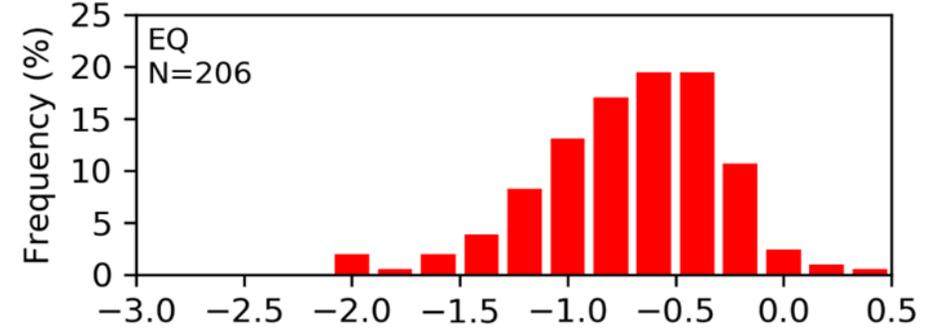
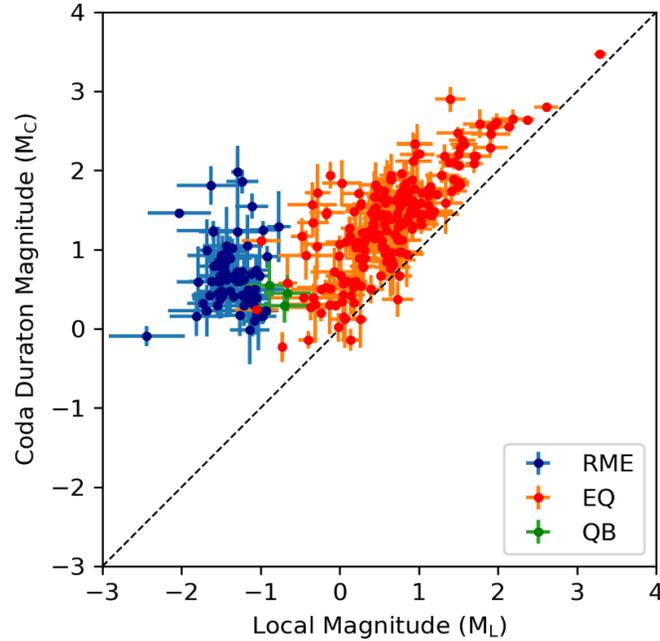
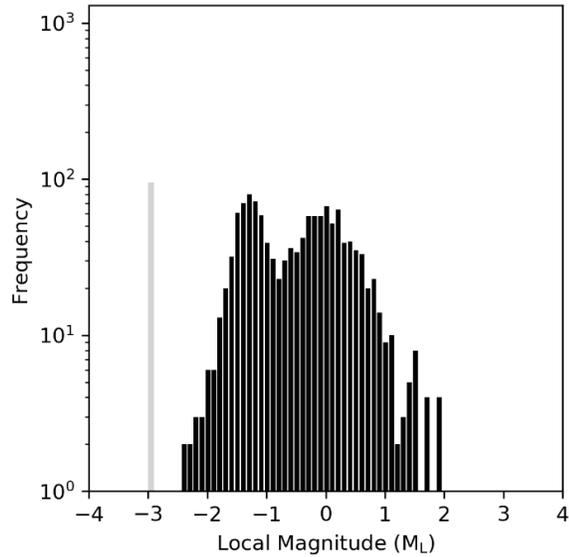
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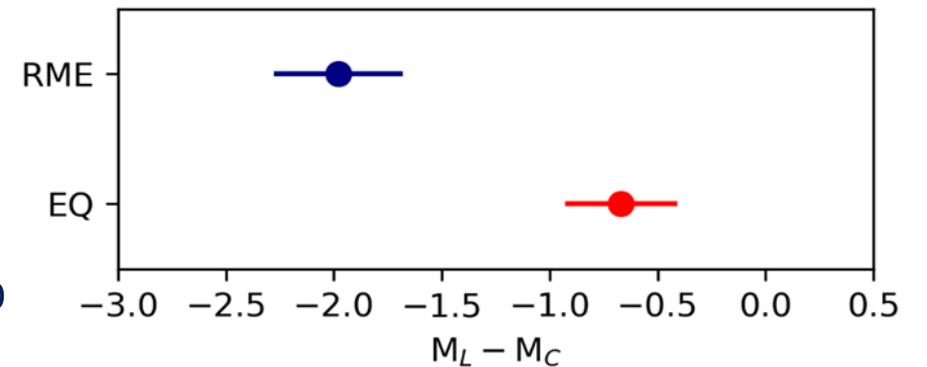
- We built 1373 events. For 284 of the events, both  $M_L$  and the coda duration magnitude ( $M_C$ ) are well constrained.
- Based on the event locations and the signal onset characters, this subset was divided into three populations:
  - 75 blasts from the Redmond Salt Mine (**RMEs**),
  - 206 tectonic earthquakes (**EQs**), and
  - 3 blasts (**QBs**) from a mine/quarry located about 8 km from the Redmond Mine.
- **We used the subset of events to test and design discriminants that are effective at local distances.**

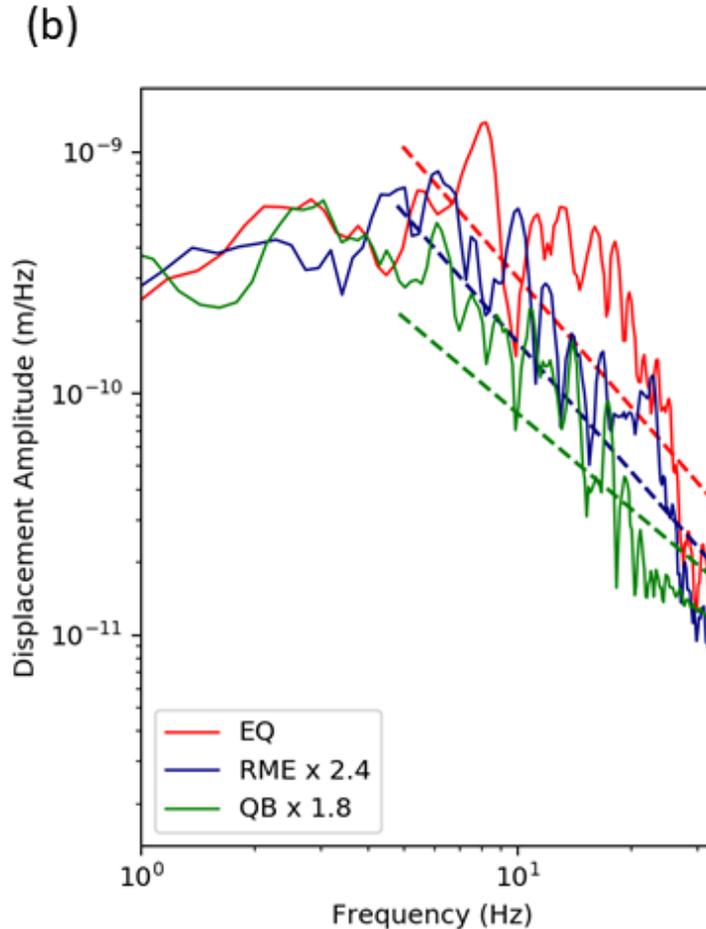
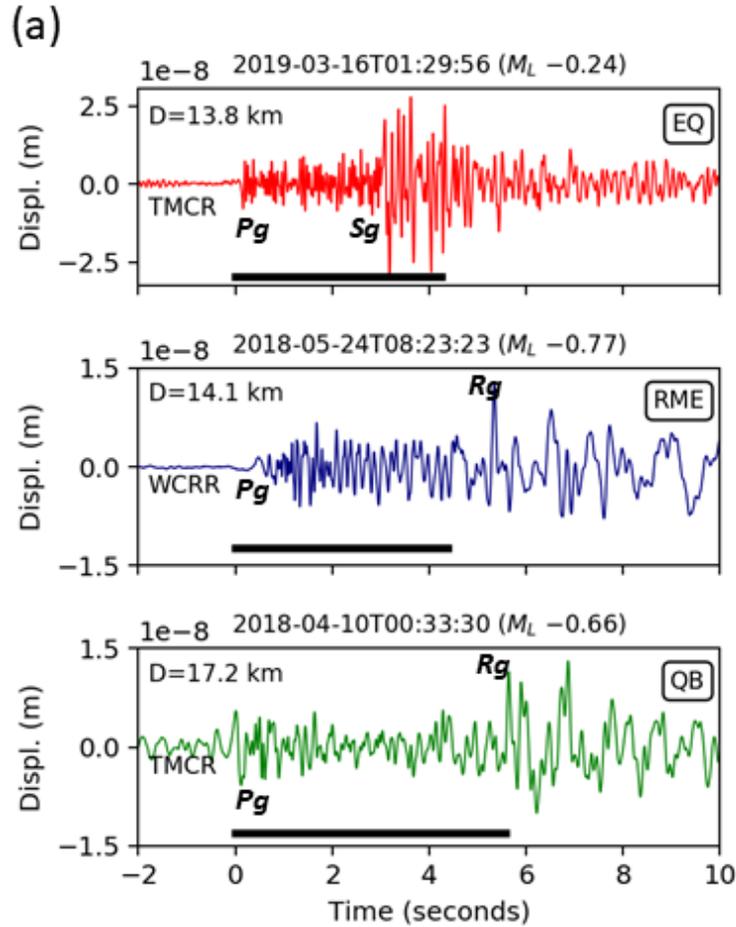




- Data points for the mining events are significantly off the 1:1 line, due to enhanced coda for the shallow mining events (Koper et al., 2016).

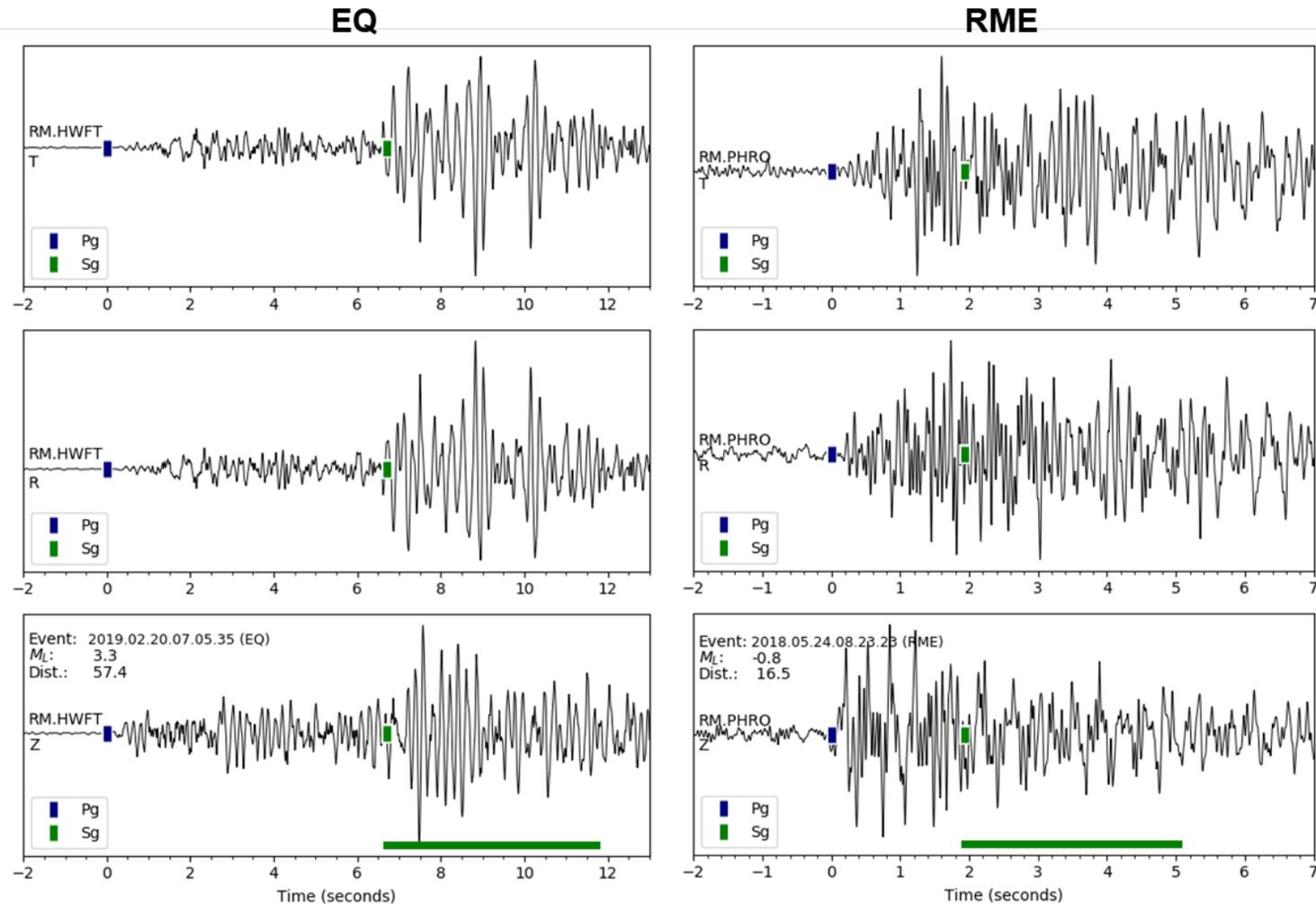
- Chi-square tests suggest that the two populations are statistically distinct.





- Differing excitations of  $Pg$ ,  $Sg$ , and  $Rg$  between the events
- Steeper fall-off slope for EQ spectrum, indicating a relative deficiency in high-frequency energy
- Dissimilarities between event classes constitute the base for discriminants investigated

## Measurement of Sg Phase Amplitudes (for Low Frequency Sg to High Frequency Sg Discriminant)



- The root mean square (RMS) of  $S_g$  amplitudes are measured in the four frequency bands 5–10 Hz, 10–15 Hz, 15–20 Hz, and 20–30 Hz.
- As a result of the Parseval's theorem, the RMS in the time domain is equivalent to the RMS in the frequency domain.

## Separation of Source Excitation, Propagation, and Site Terms

- The recorded amplitude  $A_{ij}$  for an event  $i$  recorded at a station  $j$  is expressed as in Equation 1.

$$\log A_{ij}(f) = \log EXC_i(f) + \log SITE_j(f) + \log G(r_{ij}, f) \quad (1)$$

$EXC_i(f)$ : Source excitation term for source  $i$ ;  $SITE_j(f)$ : Site term for station  $j$ ;

$G(r_{ij}, f)$ : Distance-correction term (combined effect of geometrical spreading and attenuation);  $r_{ij}$ : Distance from source  $i$  to station  $j$ .

- We parameterized the distance-correction term using a piecewise linear function (Yazd, 1993; Kintner et al., 2020). We defined series of nodes,  $r_k$ , in 5-km increment over the source-distance range of 5–100 km ( $k = 1, 2, \dots, 20$ ).
- Distance-correction term,  $G(r_{ij}, f)$ , approximated using linear interpolation:

$$G(r_{ij}, f) = G(r_k, f) + \frac{(r_{ij}-r_k)}{(r_{k+1}-r_k)} (G(r_{k+1}, f) - G(r_k, f)) \quad (2)$$

- Linearized equation for an observation at  $r_{ij}$  between  $r_k$  and  $r_{k+1}$ :

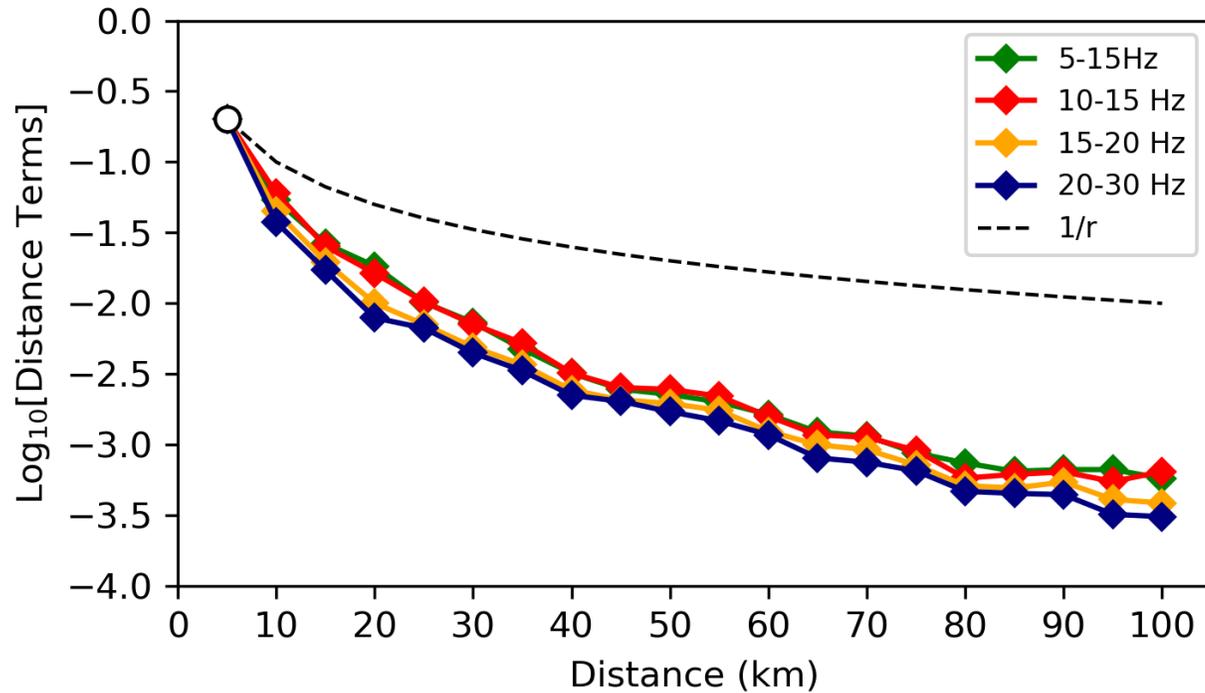
$$\log A_{ij}(f) = \log EXC_i(f) + \log SITE_j(f) + q \log G(r_k, f) + p \log G(r_{k+1}, f), \quad (3)$$

$$\text{where } p = \frac{(r_{ij}-r_k)}{(r_{k+1}-r_k)} \text{ and } q = 1 - p$$

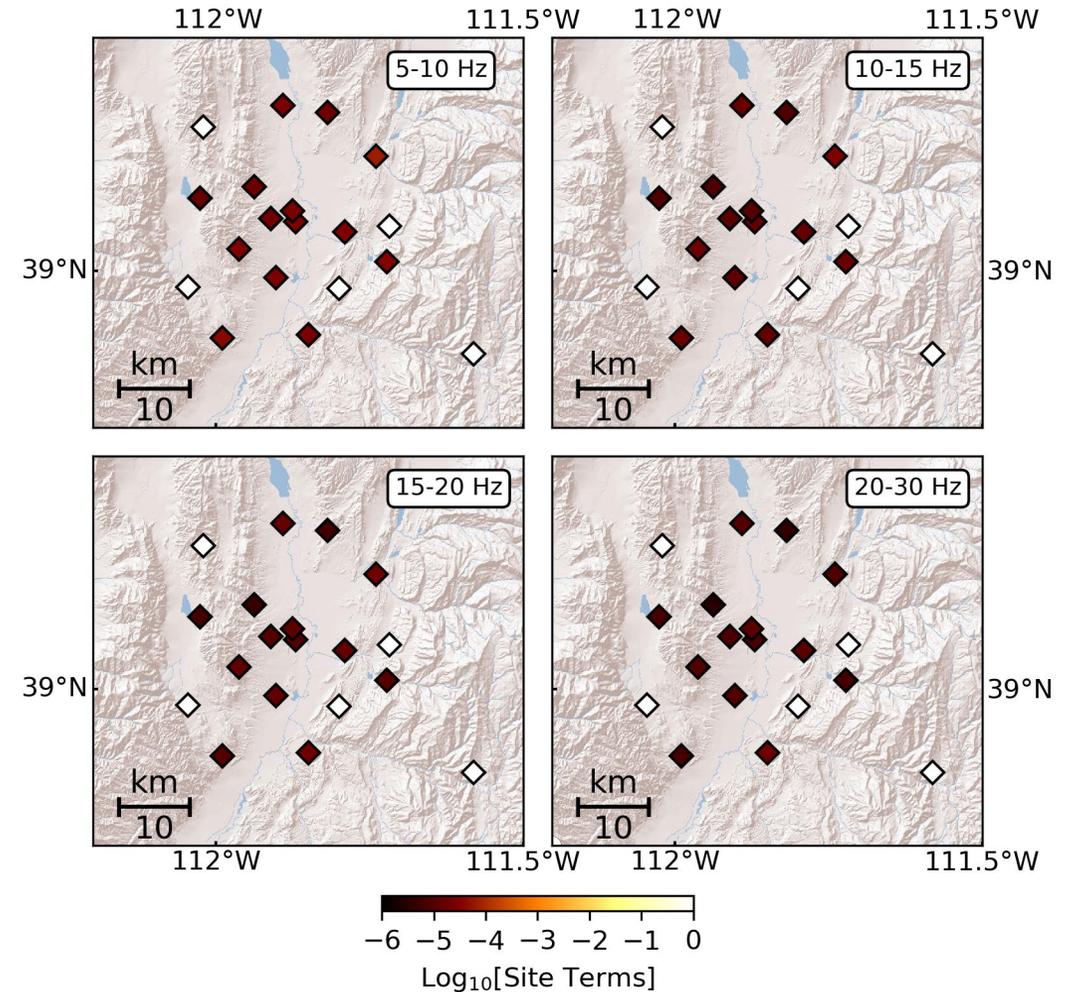
- Matrix form:  $\mathbf{a} = \mathbf{K}\mathbf{m}$  (4)

- $\mathbf{a}$ : Array of logarithm of amplitude measurements
- $\mathbf{m}$ : Array of logarithm of model parameters (source excitation terms, station site terms, and distance-correction terms at nodes 5, 10, 15, ..., 100 km)
  - 276 sources, 19 stations, and 20 distance nodes.
- Constraint: For  $r_k = 5$  km,  $G(r_k, f) = 1/r_k$  (i.e., only geometrical spreading, no attenuation)
- $\mathbf{m}$  is estimated using the SVD technique.

**Propagation and Site Terms**

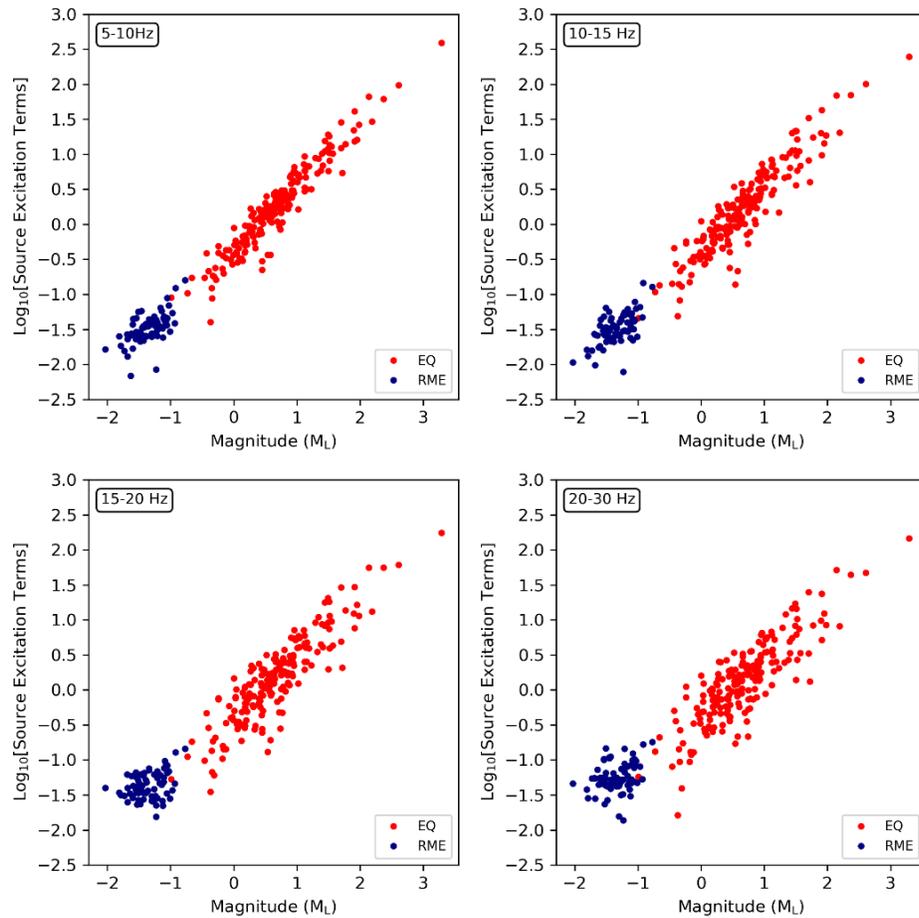


As would be expected for a frequency-dependent attenuation model, high-frequency Sg signals are slightly more attenuated than low frequencies.

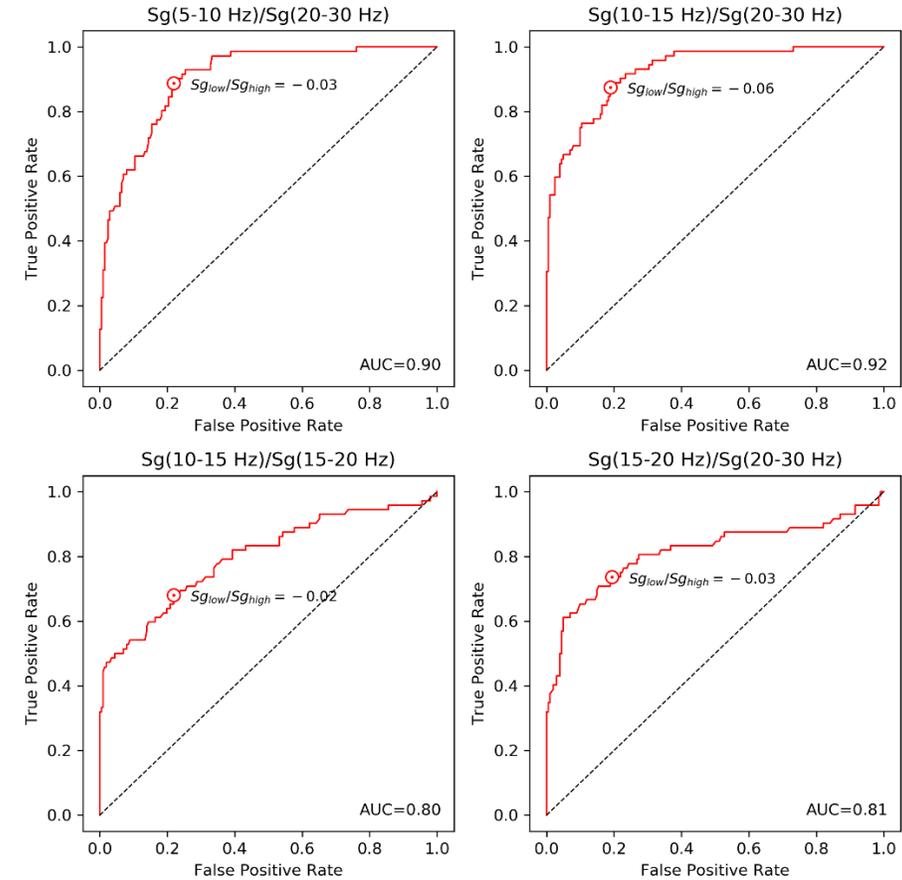


The observed damping at most stations is likely due to locations in basins.

## Source Terms and Low Frequency Sg to High Frequency Sg Discriminant

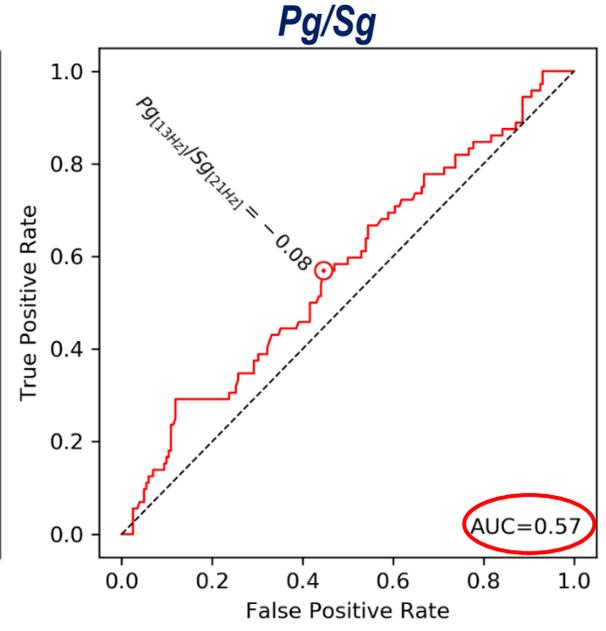
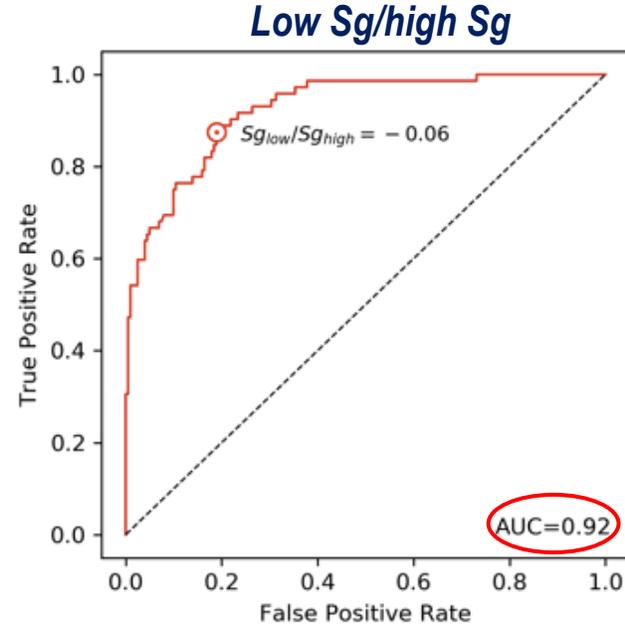
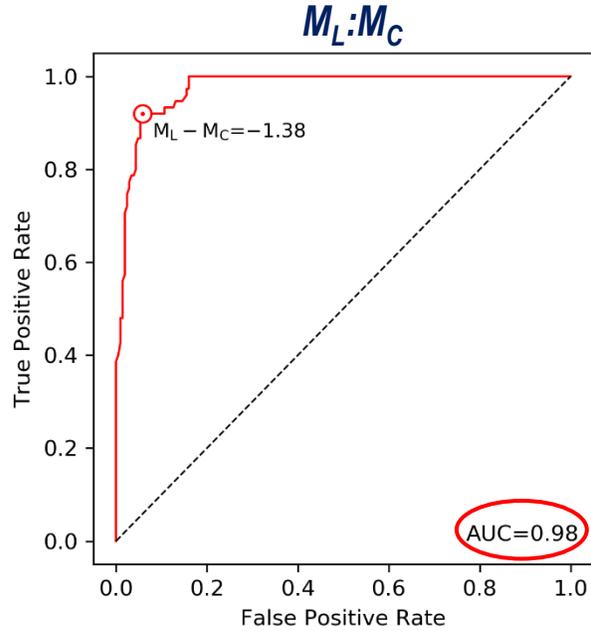
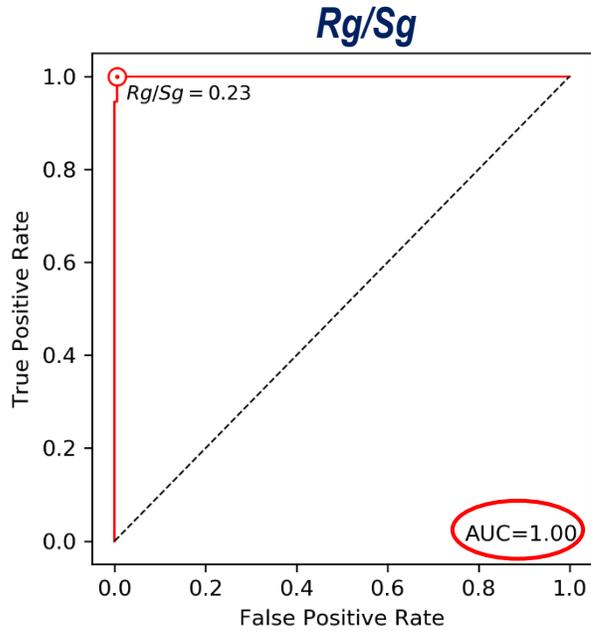


The degree of scattering of the data points increases with increasing frequency, likely due to decreasing SNR.



ROC curve for RMEs vs. EQs. The AUCs range from 0.80 to 0.92, with the highest value associated with  $Sg_{[10-15\text{Hz}]} / Sg_{[20-30\text{Hz}]}$ .

# DISCRIMINANT COMPARISON



$$LM_{min} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\Delta/2} e^{-x^2/2} dx$$

	Discriminant			
	$Rg_{[0.5-2\text{Hz}]} / Sg_{[0.5-8\text{Hz}]}$	$M_L - M_C$	$Sg_{[10-15\text{Hz}]} / Sg_{[20-30\text{Hz}]}$	$Pg/Sg$
<b>Mahalanobis Distance (<math>\Delta^2</math>)</b>	11.8	4.0	1.95	0.03
<b>Minimum Likelihood of Misclassification (<math>LM_{min}</math> in %)</b>	4.3	16.0	24.3	46.4

## **GOAL: Improve discriminant power by combining 2 or more (uncorrelated) classifiers**

- A multivariate quadratic discriminant function (QDF),  $\mathbf{D}(\mathbf{v})$ , is defined as:

$$\mathbf{D}(\mathbf{v}) = \mathbf{v}^T \mathbf{A} \mathbf{v} + \mathbf{B} \mathbf{v} + k, \quad (5)$$

where

$$\mathbf{A} = -\frac{1}{2} (\mathbf{S}_{re}^{-1} - \mathbf{S}_{eq}^{-1}) \quad (6)$$

$$\mathbf{B} = \mu_{ne}^T \mathbf{S}_{re}^{-1} - \mu_{eq}^T \mathbf{S}_{eq}^{-1} \quad (7)$$

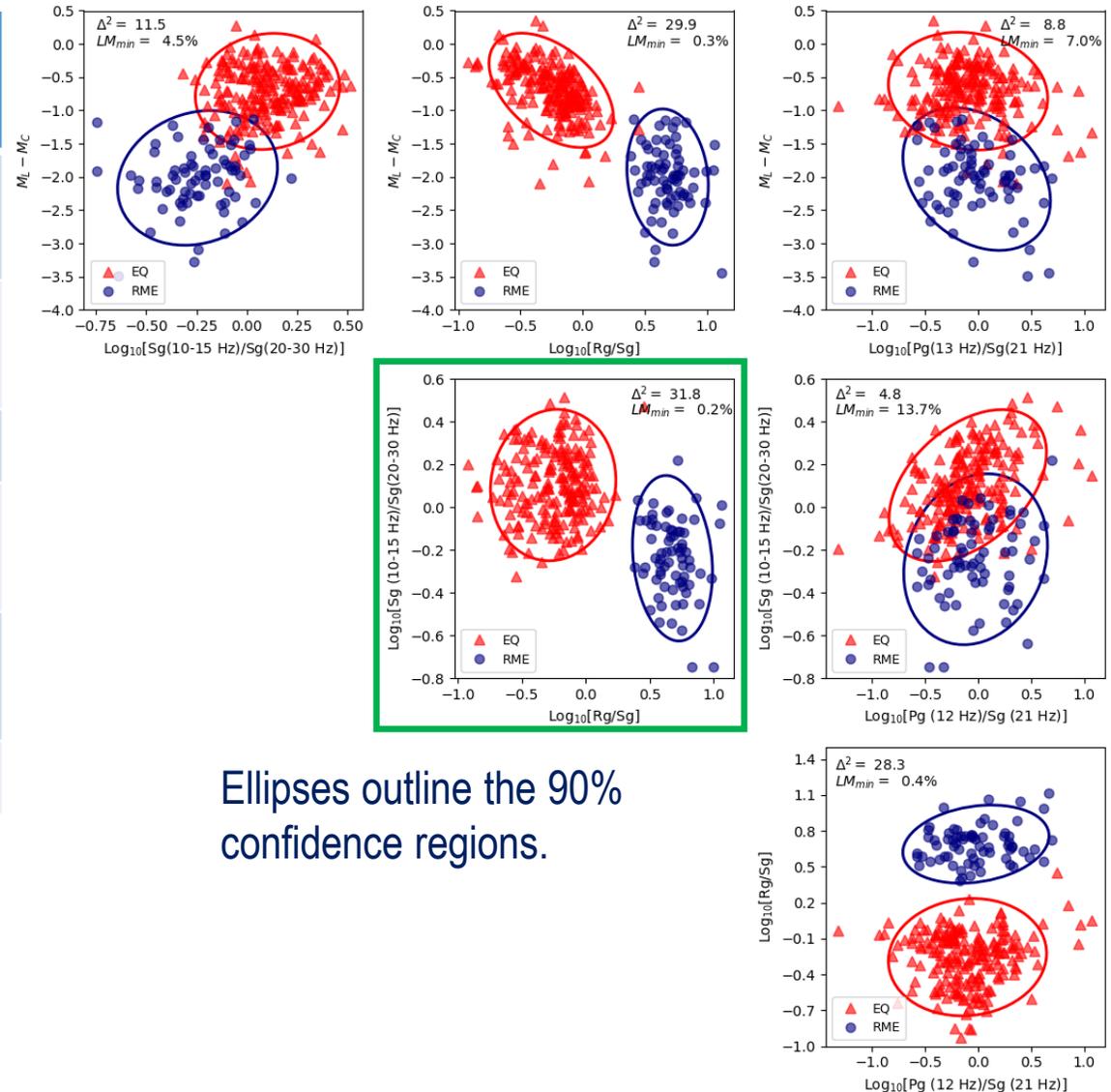
$$k = -\frac{1}{2} \left[ \ln \left( \frac{|\mathbf{S}_{re}|}{|\mathbf{S}_{eq}|} \right) + (\mu_{re}^T \mathbf{S}_{re}^{-1} \mu_{re} - \mu_{eq}^T \mathbf{S}_{eq}^{-1} \mu_{eq}) \right] \quad (8)$$

$\mathbf{v} = (v_1, \dots, v_n)^T$  is the  $n$ -dimensional vector of  $M_L - M_C$  and/or amplitudes ratios;  $\mathbf{S}_{re}$  and  $\mathbf{S}_{eq}$  are the covariance matrices of the parameters;  $\mu_{re}$  and  $\mu_{eq}$  the vector means of the parameters for the learning events for the populations  $re$  and  $eq$ , respectively.

- An event of interest with the parameter vector  $\mathbf{x} = (x_1, \dots, x_n)^T$  is classified as  $re$ -type if  $\mathbf{D}(\mathbf{x})$  is positive, and  $eq$ -type if  $\mathbf{D}(\mathbf{x})$  is negative, with  $\mathbf{D} = \mathbf{0}$  representing the classification line.

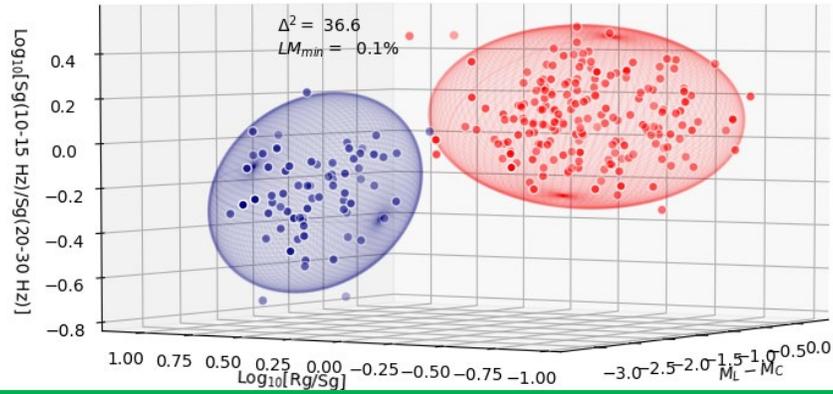
Combination	LM <sub>min</sub> (%)	LM Improvement (%)
$M_L - M_C$ and low Sg/high Sg	4.5	-0.2
$M_L - M_C$ and Rg/Sg	0.3	4
$M_L - M_C$ and Pg/Sg	7.0	-2.7
low Sg/high Sg and Rg/Sg	0.2	4.1
low Sg/high Sg and Pg/Sg	13.7	-9.4
Rg/Sg and Pg/Sg	0.4	3.9

LM improvements are relative to the best performing single classifier, Rg/Sg.

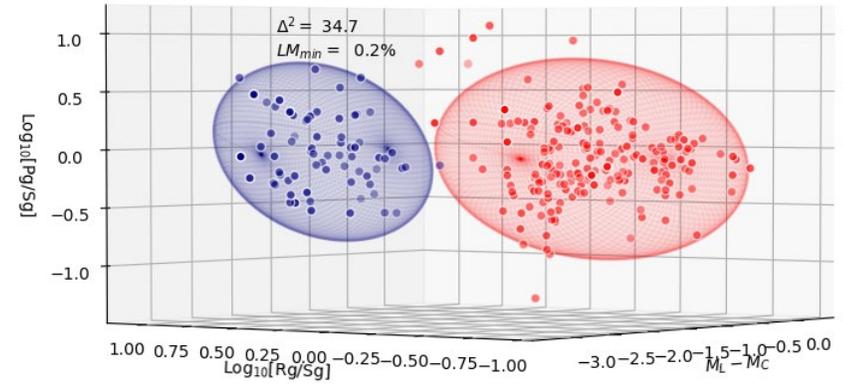


Ellipses outline the 90% confidence regions.

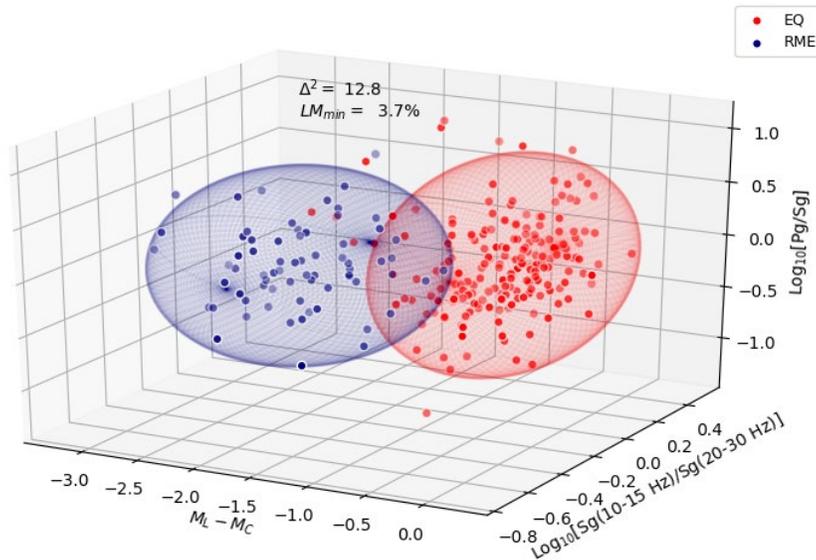
**LM Improvement: 0.1%**



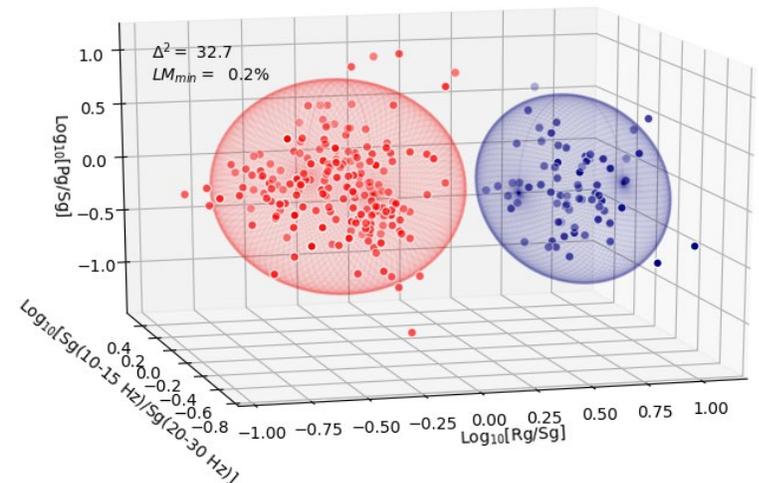
**LM Improvement: 0%**



**LM Improvement: -1.7%**



**LM Improvement: 0%**



- We tested and designed several classifiers to separate the population of mining blasts from the group of earthquakes recorded at local distances. The classifiers consist of  $M_L - M_C$ , low frequency  $Sg$  to high frequency  $Sg$ ,  $Pg/Sg$ , and  $Rg/Sg$  ratios, and different combinations of 2 or more of these classifiers.
- While the areas under the receiver operating characteristic curve (AUC) of 0.92–1.0 for  $M_L - M_C$ , low  $Sg$ /high  $Sg$ , and  $Rg/Sg$  indicate that these discriminants are very effective, the AUC of only 0.57 suggests that  $Pg/Sg$  is only slightly better than a random classifier.
- Among the individual classifiers,  $Rg/Sg$ , which is a depth discriminant, shows the lowest likelihood of misclassification ( $LM_{\min} = 4.3\%$ ) for the populations.
- To improve the discriminant power, we combined 2 and more of the discriminants by performing multivariate discriminant analyses. For the bivariate classifier, the combination of  $Rg/Sg$  with low  $Sg$ /high  $Sg$  provides the largest improvement (4.1% or  $LM_{\min} = 0.2\%$ ) over the best single discriminant, while for the best performing tri-variate classifier this improvement is 4.2% ( $LM_{\min} = 0.1\%$ ).

